H. -G M LLER, J AD M LLER, F AO

KE : E ; ; ; ; .

1. IN ROD C ION

1974), (1999), M (M 1995), (, , J - 1986), (M 1987), (E 1993).

M (1998), (1998), (2000), M J (2004),

g,

$$Y_j = g(t_j) + e_j(t_j), \qquad j = 1, \ldots, J.$$

H $(t_j)_{j=1,\ldots,J}$ $v(t_j) =$ $(e_j(t_j))$

(X, Y),

$$v(x) = E(Y^2|X = x) - [E(Y|X = x)]^2.$$

A

(2002, 2005).

(2002) (K , , 1998).

1. A

A

H , (2001)

1

A

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89069 , (E- : stamue@mathematik.uni-ulm.de).

A M

, 80523 (E- : fyao@stat.colostate.edu).

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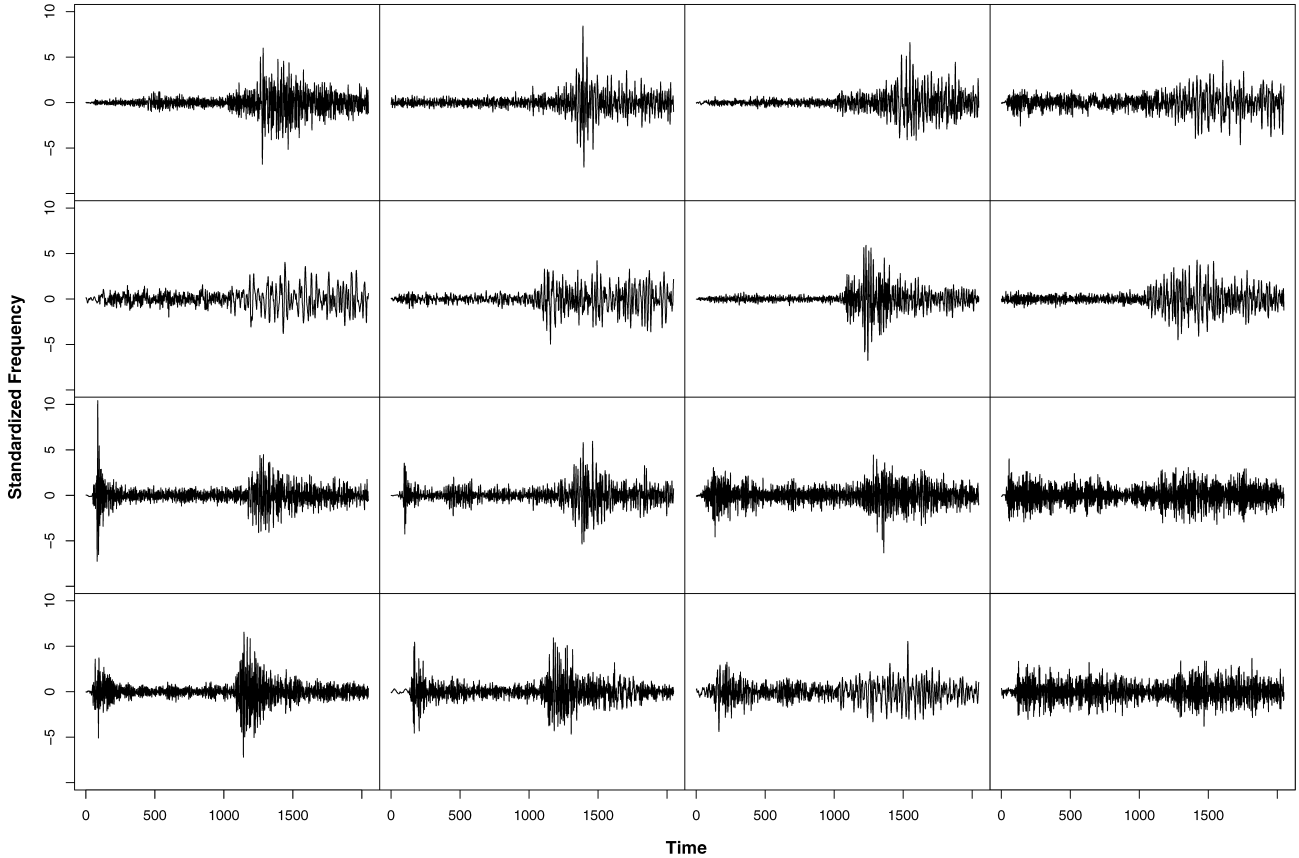

F 1. D E ( ), E (.), O. E ( ). .025

3,

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(. 1),

(. 2),

(. 3)

1.

5,

2.

1986).

(1950) (1958),

(1991;

1992; 2005; , H ,

2001; . 2003).

2. DECOMPO ING F NC IONAL DA A

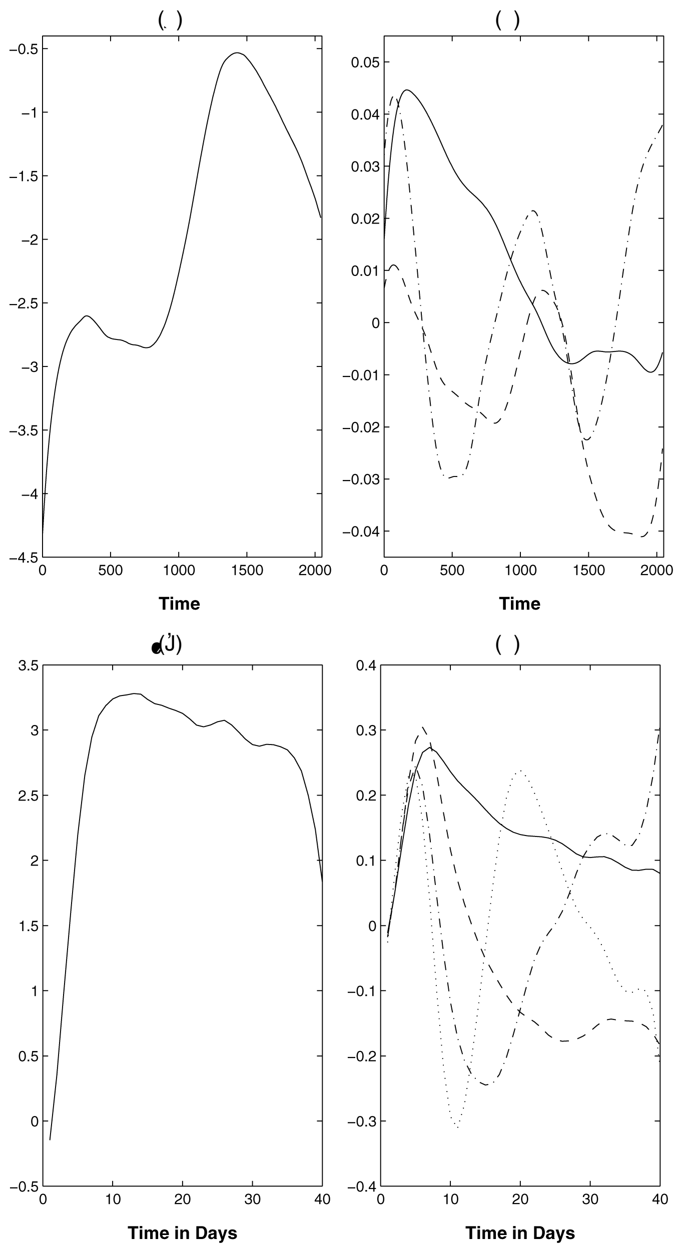
S

V

S

n S_i,

$i = 1, \ldots, n$. X_{ij}



F 2. C n F J : J P J E . E D . ( ) . ( ) E -L D ( ) . ( ) . ( ) ( ) n n n J J J J ( ) ( ) ( ). ( ) n n ( ), J- (----), (-·-·-·) J ( ) ( ). 62.8%, 23.6%, 7.8% ( ) n n ( ), J (----), (-·-·-·), (······) J ( ) 48.3%, 21.0%, 11.6%, 6.7% n .025 J ( ) ( ), ( ) ( ).

t_{ij} = $[a_1, a_2]$ S

$$X_{ij} = S_i(t_{ij}) + R_{ij}, \qquad i = 1, \ldots, n, j = 1, \ldots, m. \qquad (1)$$

R_{ij} , R_{ij} $R_{i'k}$ $i \neq i'$,

$$ER_{ij} = 0, \qquad (R_{ij}) = \sigma^2_{Rij} < \infty.$$

R_{ij} i , S $= [a_1, a_2]$. m , (A2.5). R_{ij}

, (2)

$$(Z_{ij}, Z_{ik}) = \big(V_i(t_{ij}), V_i(t_{ik})\big) = G_V(t_{ij}, t_{ik}), \quad j \neq k. \tag{9}$$

G_V , Z_{ij}. H

σ_W^2. (1998)

(2005). μ_V G_V ,

(1991).

A

, φ_k ψ_k, $k = 1, 2, \ldots,$ λ_k ρ_k $\sum_k \lambda_k < \infty$ $\sum_k \rho_k < \infty$. ξ_k $E(\xi_k) = 0$ $(\xi_k) = \lambda_k$ ζ_k $E(\zeta_k) = 0$ $(\zeta_k) = \rho_k$, μ_S μ_V ,

$$S(t) = \mu_S(t) + \sum_{k=1}^{\infty} \xi_k \varphi_k(t)$$

$$V(t) = \mu_V(t) + \sum_{k=1}^{\infty} \zeta_k \psi_k(t). \tag{10}$$

W_{ij} $\varepsilon_{ij}, i = 1, \ldots, n,\ j = 1, \ldots, m,$ $E(W_{ij}) = 0$, $(W_{ij}) = \sigma_W^2$, $P(\varepsilon_{ij} > 0) = P(\varepsilon_{ij} < 0) = \frac{1}{2}$. X_{ij}

$$X_{ij} = S_i(t_{ij}) + (\varepsilon_{ij})\{ [V_i(t_{ij}) + W_{ij}]\}^{1/2}. \tag{11}$$

S V S, λ_k ρ_k 0 ξ_k ζ_k.

3. E IMA ION OF MODEL COMPONEN

Z_{ij} , , A V_i , (7), ζ_k V, (8). A (2003), , $V(t_{ij})$ $\hat{Z}_{ij}$ μ_V ψ_k $\hat{\mu}_k$ $\hat{\psi}_k$. A $(W_{ij}) = \sigma_W^2$.

A X_{ij} $(t_{ij}), i = 1, \ldots, n, j = 1, \ldots, m,$ i j , (1) (7) A (A).

(2005) (1998).

; , A A.

, $(t_{ij}, X_{ij}),\ j = 1, \ldots, m,$ S_i;

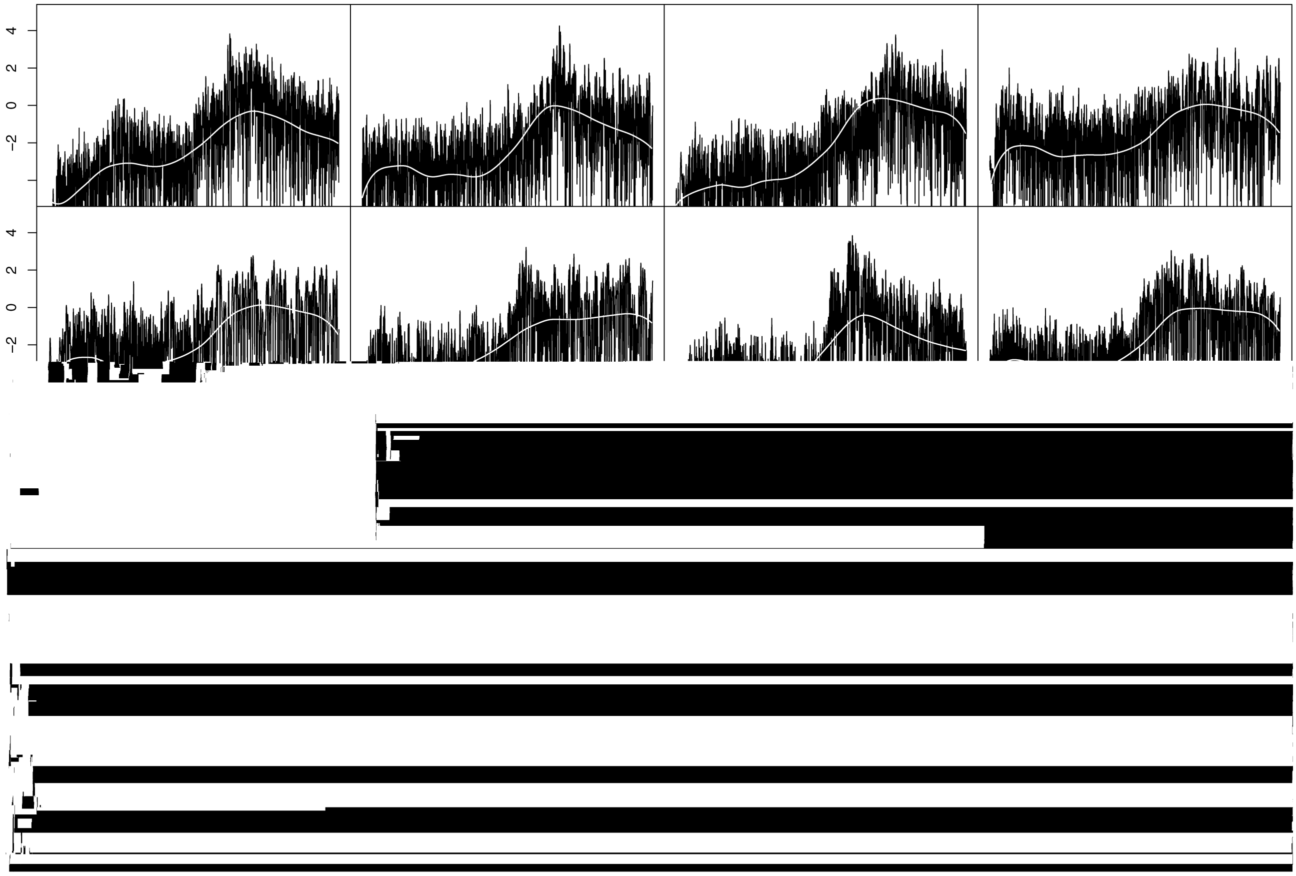
$b_{S,i}$; A A $b_{S,i}$, S. $\hat{S}_i(t_{ij})$ (A.1) A A $S_i(t_{ij})$. A , $\hat{R}_{ij} = X_{ij} - \hat{S}_i(t_{ij})$

$$\hat{Z}_{ij} = (\hat{R}_{ij}^2) = \big(X_{ij} - \hat{S}_i(t_{ij})\big)^2, \quad i = 1, \ldots, n, j = 1, \ldots, m. \tag{12}$$

, A $\hat{Z}_{ij}, i = 1, \ldots, n, j = 1, \ldots, m,$ A :

1. $\hat{Z}_{ij}$, μ_V (4) b_V ; (A.2) A A. b_V
2. E G_V (5) (A.3) , h_V, $(t_{ij}, t_{ij'})$, $t_{ij} \neq t_{ij'}$, σ_W^2; (2).
3. , ; (A.4).
4. E $(W_{ij}) = \sigma_W^2$ (2) . b_{Q_W} $\hat{\sigma}_W^2$ (A.7). A , b_{Q_W}
5. E ζ_j (8) (A.5).

M V, ;



F 3. O (R²( )) ( ) . E F P (13) ( ) E E ( ), E (.), E ( ). F 1. .025

(A.6). A (2005),

$\hat{\mu}_V$, $\hat{\psi}_k$ $\hat{\rho}_k$, $\hat{\sigma}^2_W$, $\hat{\zeta}_{ik}$. A (7), M

$$\hat{V}_i(t) = \hat{\mu}_V(t) + \sum_{k=1}^{M} \hat{\zeta}_{ik}\hat{\psi}_k(t). \qquad (13)$$

E 3.

4. A MP O IC RE L

(13) Z_{ij} (3) $\hat{Z}_{ij}$ (12) S_i. b_S S_i (A.1) A . A ; b_S $b_{S,i}$,

$b_{S,i}$, b_S, (A2.1), b_S, (A2.2); A . b_V, h_V, b_{Q_V} $\hat{\mu}_V$ (A.2), $\hat{G}_V(s,t)$ (A.3), $\hat{Q}_V(t)$ (A.7) A A. (A2.3) (A2.5).

m . A (A) ( ) A , A

Theorem 1. (A1), (A2), (1.1), (2.1), $\hat{S}_i(t)$

$$E\left(\sup_{t \in \mathcal{T}} |\hat{S}_i(t) - S_i(t)|\right) = O\left(b_S^2 + \frac{1}{\sqrt{m b_S}}\right). \qquad (14)$$

$\hat{\mu}_V(t)$ $\mu_V(t)$, $\hat{G}_V(s,t)$ $G_V(s,t)$, $\hat{\rho}_k$ $\hat{\psi}_k(t)$ ρ_k ψ_k. (A.2), (A.3), (A.4) A A . , $\hat{\sigma}_W^2$ (A.7) σ_W^2 (2) .

Theorem 2. (A1) (A8) (1.1) (2.2),

$$\sup_{t\in\mathcal{T}} |\hat{\mu}_V(t) - \mu_V(t)| = O_p\left(b_S^2 + \sqrt{\frac{1}{mb_S}} + \sqrt{\frac{1}{nb_V}}\right),$$

$$\sup_{s,t\in\mathcal{T}} |\hat{G}_V(s,t) - G_V(s,t)| = O_p\left(b_S^2 + \sqrt{\frac{1}{mb_S}} + \sqrt{\frac{1}{nh_V^2}}\right), \tag{15}$$

$$|\hat{\sigma}_W^2 - \sigma_W^2| = O_p\left(b_S^2 + \sqrt{\frac{1}{mb_S}} + \sqrt{\frac{1}{nh_V^2}} + \sqrt{\frac{1}{nb_{Q_V}}}\right).$$

ρ_k 1, $\hat{\psi}_k$

$$\sup_{t\in\mathcal{T}} |\hat{\psi}_k(t) - \psi_k(t)| \xrightarrow{p} 0, \qquad \hat{\rho}_k \xrightarrow{p} \rho_k. \tag{16}$$

$\hat{\rho}_k$ $\hat{\psi}_k$ $\sup_{t\in\mathcal{T}} |\hat{\psi}_k(t) - \psi_k(t)| = O_p(\alpha_{nk} + \alpha_{nk}^*)$ $|\hat{\rho}_k - \rho_k| = O_p(\alpha_{nk} + \alpha_{nk}^*)$, α_{nk} α_{nk}^* (.1) A , (.1) (.2).

A $\hat{V}_i$ (13) V_i, 3. $\hat{\zeta}_{ik}$ (A.5) ζ_{ik} V. .

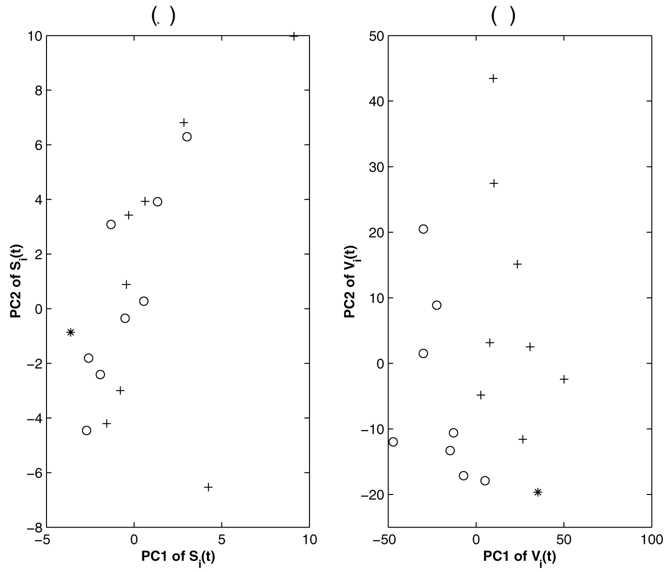
Theorem 3. (A1) (A8) (1.1) (2.2), V

$$\sup_{1\le k\le M} |\hat{\zeta}_{ik} - \zeta_{ik}| \xrightarrow{p} 0, \tag{17}$$

(13), $M = M(n) \to \infty$, $n \to \infty$. , $\hat{V}_i(t)$ V, $1 \le i \le n$,

$$\sup_{t\in\mathcal{T}} |\hat{V}_i(t) - V_i(t)| \xrightarrow{p} 0. \tag{18}$$

(16),



F 4. R ... PC2 ... PC1, O ... (+, ; ∘, ; ∗).

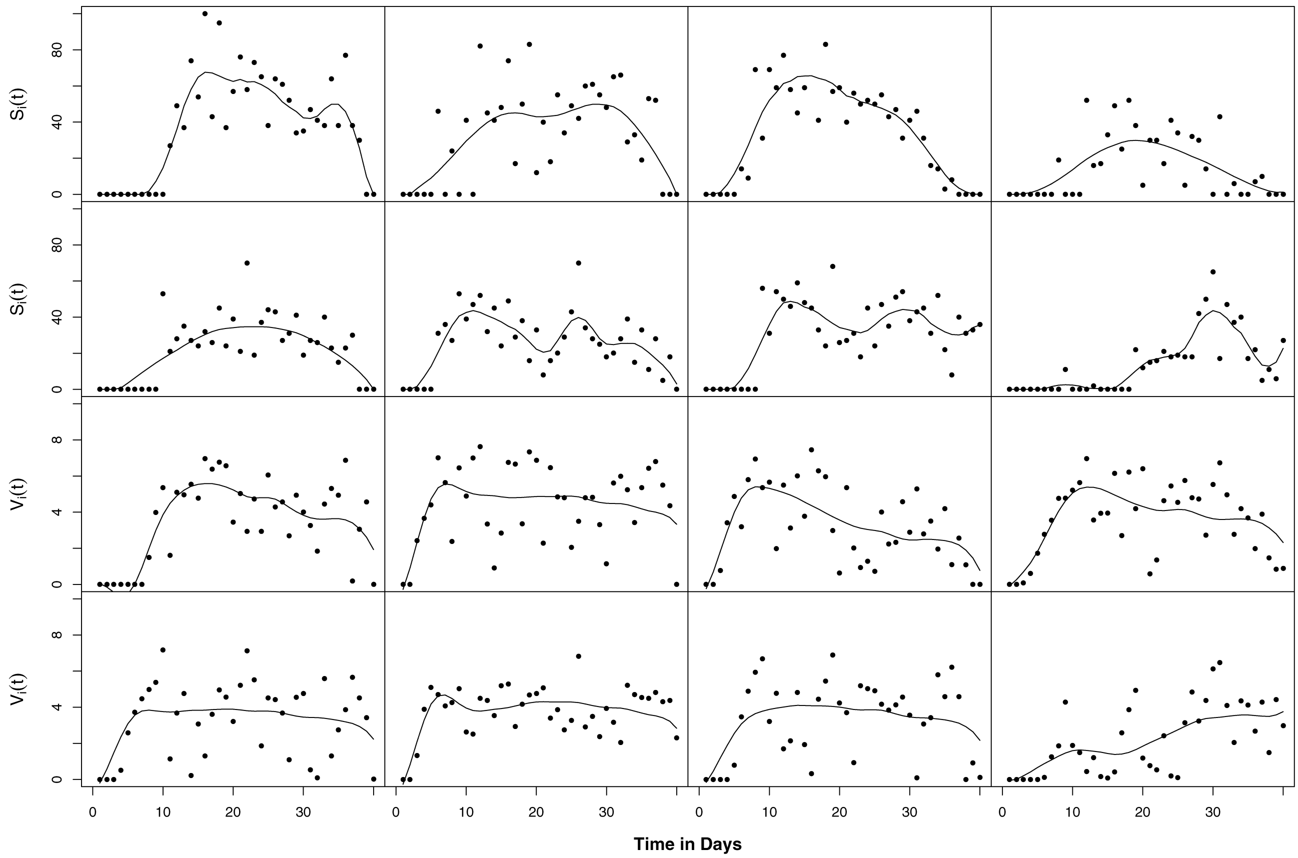
$(X_{ij} - \hat{S}_i(t_{ij}))^2$ 0, ... .001 ... $\hat{R}^2_{ij}$ ...

... V_i ... 1 ... 3. ... 1. ...

E ... $\hat{\zeta}_{ik}$, $k = 1, 2$, $i = 1, \ldots, 15$ (A.5) ... (8) ... V, ... 4( ), ... S (... 1991, ... 2003) ... 4( ). ... V_i ... S_i. ... V_i ... S, ... 4( ), ... S, ... $(\hat{\zeta}_{i1}, \hat{\zeta}_{i2})$ ... S ... 7 ... (15), ... 0 ... S, ...

$\hat{\zeta}_{ik}$, $k = 1, 2$, ... $V_i(t)$. ...

5.2 E-L D


S_i(t)
S_i(t)
V_i(t)
V_i(t)
Time in Days

APPENDIX A: ESTIMATION PROCEDURE

$\kappa_1(\cdot)$ $\kappa_2(\cdot,\cdot)$ (2.1) (2.2). $b_V = b_V(n)$ $h_V = h_V(n)$ μ_V (4) G_V (5) 1 2 A Z_{ij}.

(1996) S_i, $i = 1,\dots,n$, (t_{ij}, X_{ij}), $j = 1,\dots,m$, $b_{S,i}$

$$\sum_{j=1}^{m} \kappa_1\left(\frac{t_{ij}-t}{b_{S,i}}\right)\{X_{ij} - \beta_{i,0} - \beta_{i,1}(t - t_{ij})\}^2 \qquad (A.1)$$

$\beta_{i,0}$ $\beta_{i,1}$. $\hat{S}_i(t_{ij}) = \hat{\beta}_{i,0}(t_{ij})$. $b_{S,i}$ b_S (A2.1). μ_V, A

$$\sum_{i=1}^{n}\sum_{j=1}^{m} \kappa_1\left(\frac{t_{ij}-t}{b_V}\right)\{\hat{Z}_{ij} - \beta_0 - \beta_1(t - t_{ij})\}^2 \qquad (A.2)$$

β_0 β_1, $\hat{\mu}_V(t) = \hat{\beta}_0(t)$. $_i(t_{ij_1}, t_{ij_2}) = (\hat{Z}_i(t_{ij_1}) - \hat{\mu}_V(t_{ij_1}))(\hat{Z}_i(t_{ij_2}) - \hat{\mu}_V(t_{ij_2}))$, $G_V(s,t)$

$$\sum_{i=1}^{n}\sum_{1\le j_1 \ne j_2 \le m} \kappa_2\left(\frac{t_{ij_1}-s}{h_V}, \frac{t_{ij_2}-t}{h_V}\right) \times \{\ _i(t_{ij_1}, t_{ij_2}) - f(\beta,(s,t),(t_{ij_1}, t_{ij_2}))\}^2, \qquad (A.3)$$

$f(\beta,(s,t),(t_{ij_1}, t_{ij_2})) = \beta_0 + \beta_{11}(s - t_{ij_1}) + \beta_{12}(t - t_{ij_2})$, $\beta = (\beta_0, \beta_{11}, \beta_{12})$, $\hat{G}_V(s,t) = \hat{\beta}_0(s,t)$. $\{\rho_k, \psi_k\}_{k\ge1}$ $\{\hat{\rho}_k, \hat{\psi}_k\}_{k\ge1}$

$$\int_{\mathcal{T}} \hat{G}_V(s,t)\hat{\psi}_k(s)\,ds = \hat{\rho}_k\hat{\psi}_k(t), \qquad (A.4)$$

$\{\hat{\psi}_k\}_{k\ge1}$ (, . 2003). M ζ_{ik} (8),

$$\hat{\zeta}_{ik} = \sum_{j=2}^{m}(\hat{Z}_{ij} - \hat{\mu}_V(t_{ij}))\hat{\psi}_k(t_{ij})(t_{ij} - t_{i,j-1}), \qquad i = 1,\dots,n,\ k = 1,\dots,M. \qquad (A.5)$$

$\hat{\mu}_V^{(-i)}$ $\hat{\psi}_k^{(-i)}$ i

$$V(M) = \sum_{i=1}^{n}\sum_{j=1}^{m}\{\hat{Z}_{ij} - \hat{V}_i^{(-i)}(t_{ij})\}^2, \qquad (A.6)$$

M, $\hat{V}_i^{(-i)}(t) = \hat{\mu}_V^{(-i)}(t) + \sum_{k=1}^{M}\hat{\zeta}_{ik}^{(-i)}\hat{\psi}_k^{(-i)}(t)$ $\hat{\zeta}_{ik}^{(-i)}$ (A.5). V_i (13).

σ_W^2, G_V, $\hat{G}_V^{*}(t)$ $\{G_V(t,t) + \sigma_W^2\}$ $\hat{Q}_V(s)$, b_{Q_V}. A $\mathcal{T} = [a_1, a_2]$, $|\mathcal{T}| = a_2 - a_1$ $\mathcal{T}_1 = [a_1 + |\mathcal{T}|/4, a_2 - |\mathcal{T}|/4]$.

$$\hat{\sigma}_W^2 = \frac{1}{|\mathcal{T}_1|}\int_{\mathcal{T}_1}\{\hat{Q}_V(t) - \hat{G}_V^{*}(t)\}_+\,dt \qquad (A.7)$$

$\hat{\sigma}_W^2 > 0$ $\hat{\sigma}_W^2 = 0$. $|\mathcal{T}|/4$ (. 2003).

APPENDIX B: ASSUMPTIONS AND NOTATION

S V :

(A1) $C > 0$ S V

$$\sup_t |S^{(\nu)}(t)| < C \quad \nu = 0,1,2$$

$$\sup_t |V(t)| < C.$$

$b_{S,i} = b_{S,i}(n)$, $b_V = b_V(n)$, $h_V = h_V(n)$, $b_{Q_V} = b_{Q_V}(n)$ $\hat{S}_i$ (A.1), $\hat{\mu}_V$ (A.2), $\hat{G}_V$ (A.3), $\hat{Q}_V(t)$ (A.7). n m :

(A2.1) $b_{S,i}$, b_S c_1 c_2, $0 < c_1 < \inf_i b_{S,i}/b_S \le \sup_i b_{S,i}/b_S < c_2 < \infty$.

(A2.2) $m \to \infty$, $b_S \to 0$, $mb_S^2 \to \infty$.

(A2.3) $b_V \to 0$, $b_{Q_V} \to 0$, $nb_V^4 \to \infty$, $nb_{Q_V}^4 \to \infty$, $\limsup_n n \times b_V^6 < \infty$, $\limsup_n nb_{Q_V}^6 < \infty$.

(A2.4) $h_V \to 0$, $nh_V^6 \to \infty$, $\limsup_n nh_V^8 < \infty$.

(A2.5) $\limsup_n n^{1/2}b_V m^{-1} < \infty$, $\limsup_n n^{1/2}b_{Q_V}m^{-1} < \infty$, $\limsup_n n^{1/2}h_V m^{-1} < \infty$.

$\{t_{ij}\}_{i=1,\dots,n;j=1,\dots,m}$

i $j = 1,\dots,m-1$, $t_{ij} < t_{i,j+1}$ f $\int_{\mathcal{T}} f(t)\,dt = 1$ $\inf_{t\in\mathcal{T}} f(t) > 0$ t_{ij} $t_{ij} = F^{-1}(\frac{j-1}{m-1})$, F^{-1} $F(t) = \int_{a_1}^{t} f(s)\,ds$.

t_{ij} i, c_1 c_2 f_i $0 < c_1 < \inf_i \inf_{t\in\mathcal{T}} f_i(t) < \sup_i \sup_{t\in\mathcal{T}} f_i(t) < c_2$. N_i i $m \to \infty$ c_1 c_2, $0 < c_1 < \inf_i \frac{N_i}{m} < \sup_i \frac{N_i}{m} < c_2 < \infty$; $N_i = m$, m

n = $\{t_{ij} - t_{i,j-1} : j = 2, \ldots, m\}$,

(A3) $n = O(m^{-1})$, $n, m \to \infty$.

A X_{ij} Z_{ij} $t \in \mathcal{T}$,

(A4) $_j E[X_{ij}^4] < \infty$ $_j E[Z_{ij}^4] < \infty$.

[redacted] H H (1989). $(f \otimes g)(h) = \langle f, h\rangle g$ $f, g, h \in H$, H H H $F \equiv \sigma_2(H)$, $\langle T_1, T_2\rangle_F =$ $(T_1 T_2^*) = \sum_j \langle T_1 u_j, T_2 u_j\rangle_H$ $\|T\|_F^2 = \langle T, T\rangle_F$, $T_1, T_2, T \in F$, T_2^* T_2, $\{u_j : j \geq 1\}$ H. $\mathbf{G}_V$ (6) $\hat{\mathbf{G}}_V$, [redacted] G_V (5), $\hat{G}_V$ (A.3), H $\mathcal{I}_i = \{j : \rho_j = \rho_i\}$ $\mathcal{I}' = \{i : |\mathcal{I}_i| = 1\}$, $|\mathcal{I}_i|$ $\mathcal{I}_i$. $\mathbf{P}_j^V = \sum_{k\in\mathcal{I}_j} \psi_k \otimes \psi_k$ $\hat{\mathbf{P}}_j^V = \sum_{k\in\mathcal{I}_j} \hat{\psi}_k \otimes \hat{\psi}_k$ H $\{\psi_k : k \in \mathcal{I}_j\}$ $\{\hat{\psi}_k : k \in \mathcal{I}_j\}$. j,

$$\delta_j^V = \frac{1}{2} \quad \{|\rho_l - \rho_j| : l \notin \mathcal{I}_j\}, \qquad (\ .1)$$

$\Lambda_{\delta_j^V} = \{z \in \mathcal{C} : |z - \rho_j| = \delta_j^V\}$, $\mathcal{C}$. $\mathbf{G}_V$ $\hat{\mathbf{G}}_V$ $\mathbf{R}_V$ $\hat{\mathbf{R}}_V$, $\mathbf{R}_V(z) = (\mathbf{G}_V - zI)^{-1}$ $\hat{\mathbf{R}}_V(z) = (\hat{\mathbf{G}}_V - zI)^{-1}$.

$$A_{\delta_j^V} = \quad \{\|\mathbf{R}_V(z)\|_F : z \in \Lambda_{\delta_j^V}\} \qquad (\ .2)$$

$M = M(n)$, $V(t)$, $\hat{V}_i(t) = \hat{\mu}_V(t) + \sum_{m=1}^{M(n)} \hat{\zeta}_{im}\hat{\psi}_m(t)$ (13). $\|\pi\|_\infty = {}_{t\in\mathcal{T}} |\pi(t)|$ $\pi(\cdot)$ $\mathcal{T}$. μ_V ψ_j $M = M(n)$ n m $n \to \infty$,

(A5) $\tau_n = \sum_{j=1}^M (\delta_j^V A_{\delta_j^V} \|\psi_j\|_\infty)/(\sqrt{n}h_V^2 - A_{\delta_j^V}) \to 0$, $M = M(n) \to \infty$;

(A6) $\sum_{j=1}^M \|\psi_j\|_\infty = o(\quad\{\sqrt{n}b_V, \sqrt{m}\})$ $\sum_{j=1}^M \|\psi_j\|_\infty \times \|\psi_j'\|_\infty = o(m)$.

(A5) (A6) ψ_j j . $\theta_m = b_S^2 + (\sqrt{m}b_S)^{-1}$, V :

(A7) $E\{[\quad_{t\in\mathcal{T}} |V(t) - V^{(M)}(t)|]^2\} = o(n)$, $V^{(M)}(t) = \mu_V(t) + \sum_{k=1}^M \zeta_k\psi_k(t)$.

(A8) $1 \leq i \leq n$, $\theta_m \sum_{k=1}^M \|\psi_k\|_\infty^2 = o_p(1)$ $\gamma_n = \sum_{k=1}^M (\delta_k^V A_{\delta_k^V} \|\psi_k\|_\infty)/(\theta_m^{-1} - A_{\delta_k^V}) \to 0$ $n \to \infty$.

$t = t_{ij}, t_1 = t_{ij_1}$, $t_2 = t_{ij_2}$, i, j, j_1, j_2, $g(x; t)$ X_{ij} $g_2(x_1, x_2; t_1, t_2)$ (X_{ij_1}, X_{ij_2}). $f(z; t)$ $f_2(z_1, z_2; t_1, t_2)$ Z_{ij} (Z_{ij_1}, Z_{ij_2}). $g(\cdot; t), f(\cdot; t), t \in \mathcal{T}$, $g_2(\cdot; t_1, t_2), f_2(\cdot; t_1, t_2), t_1, t_2 \in \mathcal{T}$, :

(1.1) $(d^2/dt^2)g(x; t)$ $(d^2/dt^2)f(z; t)$ $\times \mathcal{T}$.

(1.2) $(d^2/dt_1{}^1 dt_2{}^2)g_2(x_1, x_2; t_1, t_2)$ $(d^2/dt_1{}^1 dt_2{}^2)f_2(z_1, z_2; t_1, t_2)$ $^2 \times \mathcal{T}^2$, $_1 + {}_2 = 2, 0 \leq {}_1, {}_2 \leq 2$.

[redacted] $\kappa_1 : \to$ $\kappa_2 : {}^2 \to$ $\kappa_1(u)$ $\kappa_2(u, v)$ $\chi_1(t) = \int e^{-iut}\kappa_1(u)\,du$ $\chi_2(t, s) = \int e^{-(iut+ivs)}\kappa_2(u, v)\,du\,dv$. :

(2.1) [redacted] κ_1 , $\|\kappa_1\|^2 = \int \kappa_1^2(u)\,du < \infty$, . $\chi_1(t)$ $\int |\chi_1(t)|\,dt < \infty$.

(2.2) K κ_2 , $\|\kappa_2\|^2 = \int\int \kappa_2^2(u, v)\,du\,dv < \infty$, κ_2 [redacted] 0 u v. , $\chi_2(t, s)$, $\int\int |\chi_2(t, s)|\,dt\,ds < \infty$.

APPENDI C: PROOF

P n 1

W S V, $= {}_1 \times {}_2$. E^* ^{2} . (. ., S V), $\omega_1 \in {}_1$. ω_1, $R_j = R_j(\omega_1)$ i R_{ij} (1) $_2$ j, $E^*(R_j) = 0$ $_j E^*(R_j^2) < C_1$ C_1. (A1) 3 2 [redacted] (1979) [redacted] [redacted]

$$E^*\Big(\quad_{t\in\mathcal{T}} |\hat{S}(t) - S(t)|\Big)(\omega_1) = O\Big(b_S^2 + \frac{1}{\sqrt{m}b_S}\Big)(\omega_1).$$

ω_1, $|S^{(\nu)}(\omega_1)|, \nu = 0, 1, 2$, $|V(\omega_1)|$. (A1), ω_1;

$$\quad_{\omega_1 \in {}_1} E^*\Big(\quad_{t\in\mathcal{T}} |\hat{S}(t) - S(t)|\Big)(\omega_1) = O\Big(b_S^2 + \frac{1}{\sqrt{m}b_S}\Big),$$

(14).

.1. V $\{t_{ij}, Z_{ij}\}$ (. ., [redacted]) $\tilde{\mu}_V, \tilde{G}_V, \tilde{\sigma}_W^2, \tilde{\psi}_k, \tilde{\rho}_k$, $\tilde{\zeta}_{ik}$, (A.2), (A.3), (A.4), (A.5)

$$\alpha_{nk} = \frac{\delta_k^V A_{\delta_k^V}}{\sqrt{n}h_V^2 - A_{\delta_k^V}} \qquad \alpha_{nk}^* = \frac{\delta_k^V A_{\delta_k^V}}{\theta_m^{-1} - A_{\delta_k^V}}, \qquad (\ .1)$$

$\theta_m = b_S^2 + (\sqrt{m}b_S)^{-1}$ δ_k^V $A_{\delta_k^V}$ (.1) (.2).

Lemma C.1. (A2.1) (A2.3), (A3) (A5), (1.1) (2.2),

$$\quad_{t\in\mathcal{T}} |\tilde{\mu}_V(t) - \mu_V(t)| = O_p\Big(\frac{1}{\sqrt{n}b_V}\Big)$$

$$\quad_{s,t\in\mathcal{T}} |\tilde{G}_V(s, t) - G_V(s, t)| = O_p\Big(\frac{1}{\sqrt{n}h_V^2}\Big). \qquad (\ .2)$$

, , ρ_k 1, $\hat{\psi}_k$

$$\sup_{t \in \mathcal{T}} |\tilde{\psi}_k(t) - \psi_k(t)| = O_p(\alpha_{nk}) \qquad (.3)$$

$$|\tilde{\rho}_k - \rho_k| = O_p(\alpha_{nk}),$$

$\alpha_{nk} \to 0$ $n \to \infty$, k , α_{nk} (.1), $O_p(\cdot)$ (.3) $1 \le k \le M$. A - (.2),

$$\sup_{t \in \mathcal{T}} |\tilde{\sigma}_W^2(t) - \sigma_W^2(t)| = O_p\left(\left\{ \frac{1}{\sqrt{n}h_V^2}, \frac{1}{\sqrt{n}b_{Q_V}} \right\} \right). \qquad (.4)$$

(A1) (A7) (1.1) (2.2),

$$\sup_{1 \le k \le M} |\tilde{\zeta}_{ik} - \zeta_{ik}| \xrightarrow{p} 0 \qquad (.5)$$

$$\sup_{t \in \mathcal{T}} \left| \sum_{k=1}^{M} \tilde{\zeta}_{ik} \tilde{\psi}_k(t) - \sum_{k=1}^{\infty} \zeta_{ik} \psi_k(t) \right| \xrightarrow{p} 0,$$

M $M = M(n) \to \infty$ $n \to \infty$.

Proof of Lemma C.1. (.2), (.3), (.5) 1 1 (2006). $\tilde{\sigma}^2_W$nd20.10 8.9664 268med1 Tfϑ-0.1.00 -0.6002 TDϑ(β)Tjϑ0 (

$$+ \sup_{t \in \mathcal{T}} \left\{ \left| \sum_{k=1}^{M} \tilde{\zeta}_{ik} \tilde{\psi}_k(t) - \sum_{k=1}^{\infty} \zeta_{ik} \psi_k(t) \right| \right\}$$

$$\equiv Q_{i1}(n) + Q_{i2}(n),$$

$Q_{i1}(n) \xrightarrow{p} 0$ $Q_{i2}(n) \xrightarrow{p} 0$. A (.5), $Q_{i2}(n) \xrightarrow{p} 0$ (A1) (A7), $Q_{i2}(n) = O(\vartheta_{in}^{(2)})$, $O(\cdot)$ $1 \le i \le n$. $Q_{i1}(n)$,

$$Q_{i1}(n) \le \sup_{t \in \mathcal{T}} \left\{ \sum_{k=1}^{M} |\hat{\zeta}_{ik} - \tilde{\zeta}_{ik}| \cdot |\hat{\psi}_k(t)| + \sum_{k=1}^{M} |\tilde{\zeta}_{ik}| \cdot |\hat{\psi}_k(t) - \tilde{\psi}_k(t)| \right\}. \quad (.11)$$

(.10), (.11)

$$C_1 \theta_{im} \sum_{k=1}^{M} \|\psi_k\|_\infty^2 + \tilde{\gamma}_n \left\{ C_2 + \sum_{j=2}^{m} |Z_{ij}|(t_{ij} - t_{i,j-1}) \right\} \xrightarrow{p} 0.$$

(.11) $O_p\{\sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} E|\zeta_{ik}|/(\theta_m^{-1} - A_{\delta_k^V})\}$. A $E\{\sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} E|\zeta_{ik}|/(\theta_m^{-1} - A_{\delta_k^V})\} \le \sum_{k=1}^{M} \delta_k^V A_{\delta_k^V} \sqrt{\rho_k}/(\theta_m^{-1} - A_{\delta_k^V}) \le \gamma_n$, $\rho_k \to 0$, 0 $Q_{i1}(n) = O_p(\vartheta_{in}^{(1)})$, $O_p(\cdot)$ i. (18) $\sup_{t \in \mathcal{T}} |\hat{V}_i(t) - V_i(t)| = O_p(\vartheta_{in}^{(1)} + \vartheta_{in}^{(2)})$, $O_p(\cdot)$ $1 \le i \le n$.

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