

1,2, 1, 1, 1, 2

¹ ep r en of nfor on ene, hoo of he ene n d A ,
² bor or of he ero ene, K r n ene e,
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Abstract. n e p e n o p o n e n n (A) h n p r
 p p o n n d h e d o f n n n e p r o e n n e v e r A
 e m n o r h h d e b e e n o n d e v h e e k o n o f o e
 p r o b b e n f n o n o w e v e r h e e k o f d e p d h
 e h e o v e h e e A o r h e p e f o r h e e n
 e r e h p e r n d b n o r e o e x n h p p e r,
 o r n o h e o n e b d h n p r n p e n d b m n h e e x n
 r x v e r n n f o r o n , w e p r o p o e
 o n e b h n A e m n o r h o n h e e f e n f o
 h o w n b h e e n d o e x p e r e n h o r p r o p o e d e m
 n o r h w o r k e d e n d o n h e A p r o b e w h h o h p e r
 n d b n o r e n d o p e r f o r h e e x e n d e n f o x n d
 A o r h

$$s \quad x \quad A s$$

$$y \quad Wx, \quad x \in \mathbb{R}^m, \quad y \in \mathbb{R}^n, \quad W \in \mathbb{R}^{m \times n},$$

$$A \quad W$$

$$m \quad n \quad A \quad n \times n$$

$$ff$$

$$D \quad -H y - \sum_{i=1}^n \int p \quad y_i \cdot W \quad p_i y_i dy_i,$$

$$H y \quad - \int p y \quad p y dy \quad y \quad p_i y_i \quad p \quad y_i \cdot W$$

$$y \quad Wx$$

h wor^k w p p o r e d b h e r e n e o n d o n o f h n f o r r o e
 60471054.

$$\begin{aligned}
 & \dots p_i y_i \dots \\
 & \dots S \dots \\
 & \dots X \dots \\
 & \dots L \dots \\
 & \dots X, Y, \dots \\
 & \dots X, Y \dots \\
 & \dots X, Y \dots
 \end{aligned}$$

$$\begin{aligned}
 & \dots V_{n,p} \dots \\
 & \dots V_{n,p}, \mathbb{R}^n \dots \\
 & \dots V_{n,n} \dots O_n \dots \\
 & \dots F \mathbf{Z} \dots \\
 & \dots \mathbf{Z} \in O_n \dots
 \end{aligned}$$

$$\begin{aligned}
 & \nabla F = F - \mathbf{Z} F^T \mathbf{Z}, \\
 & \dots F \mathbf{Z} \dots \mathbf{Z} \dots \\
 & \dots W \dots O_n \dots \\
 & \dots E_{xx}, E_{xx} \dots
 \end{aligned}$$

$$\mathbf{I}_n \text{ is } n \times n, \quad \mathbf{W} \text{ is } n \times p, \quad \mathbf{y} \text{ is } n \times 1, \quad \mathbf{W}\mathbf{x} \text{ is } n \times 1.$$

$$E\mathbf{y} \text{ is } n \times 1, \quad E\mathbf{y}\mathbf{y}^T \text{ is } n \times n.$$

$$\mathbf{I}_n \text{ is } n \times n.$$

$$\mathbf{I}_n - E\mathbf{y}\mathbf{y}^T - \mathbf{W}E\mathbf{x}\mathbf{x}^T\mathbf{W}^T - \mathbf{W}\mathbf{W}^T.$$

$$\mathbf{W}\mathbf{W}^T \text{ is } n \times n, \quad \mathbf{W} \text{ is } n \times p, \quad \mathbf{x} \text{ is } p \times 1, \quad \mathbf{y} \text{ is } n \times 1.$$

$$\mathbf{W} \text{ is } n \times p, \quad \mathbf{W} \text{ is } n \times p, \quad \mathbf{W} \text{ is } n \times p, \quad \mathbf{W} \text{ is } n \times p.$$

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$$p_{super} u = \frac{1}{\pi} \mathbf{x}^T u, \quad p_{sub} u = p_{N(1,1)} u - p_{N(-1,1)} u,$$

$$\mu, \sigma^2, p_{N(\mu, \sigma^2)} u \text{ is } n \times 1, \quad \mathbf{x} \text{ is } p \times 1, \quad \mathbf{y} \text{ is } n \times 1.$$

$$p_{sub} u \text{ is } n \times 1, \quad p_{super} u \text{ is } n \times 1, \quad n - p \text{ is } n \times 1.$$

$$\mathbf{V} = [v_1, v_2, \dots, v_n]^T \text{ is } n \times 1, \quad \mathbf{y} \text{ is } n \times 1, \quad \mathbf{W}\mathbf{x} \text{ is } n \times 1.$$

$$v_i = \mathbf{x}^T \mathbf{y}_i, \quad i = 1, \dots, p.$$

$$v_i = \mathbf{x}^T \mathbf{y}_i - y_i, \quad i = p + 1, \dots, n.$$

$$\mathbf{W} \text{ is } n \times p, \quad \mathbf{J} \text{ is } n \times n, \quad \mathbf{W} \text{ is } n \times p.$$

$$\mathbf{J} = -\mathbf{W} - \mathbf{V}\mathbf{x}^T.$$

$$\mathbf{W} \text{ is } n \times p, \quad \mathbf{W} \text{ is } n \times p, \quad \mathbf{W} \text{ is } n \times p, \quad \mathbf{W} \text{ is } n \times p.$$

$$\Delta \mathbf{W} = -\eta \mathbf{J} - \mathbf{W}\mathbf{J}^T \mathbf{W} - \eta \mathbf{V}\mathbf{x}^T - \mathbf{W}\mathbf{x}\mathbf{V}^T \mathbf{W},$$

$$\eta > 0 \text{ is } n \times 1, \quad \mathbf{W} \text{ is } n \times p, \quad \mathbf{W} \text{ is } n \times p, \quad \mathbf{W} \text{ is } n \times p.$$

Let J be the Jacobian of the transformation $(x, y) \rightarrow (u, v)$. Then the joint density of (u, v) is given by

$$f(u, v) = \frac{1}{|J|} f(x(u, v), y(u, v)).$$

where $x(u, v)$ and $y(u, v)$ are the inverse transformation functions.

Let X and Y be independent standard normal random variables. Then the joint density of (X, Y) is given by

$$f(x, y) = \frac{1}{2\pi} e^{-\frac{x^2 + y^2}{2}}.$$

Let U and V be independent standard normal random Variables. Then the joint density of (U, V) is given by

$$f(u, v) = \frac{1}{2\pi} e^{-\frac{u^2 + v^2}{2}}.$$

3.1 On Separating Mixed Super-Gaussian and Sub-Gaussian Sources

Let E and F be independent standard normal random Variables. Then the joint density of (E, F) is given by

$$f(e, f) = \frac{1}{2\pi} e^{-\frac{e^2 + f^2}{2}}.$$

Let χ^2 be a chi-squared random Variable with γ degrees of freedom. Then the density of χ^2 is given by

$$f(\chi^2) = \frac{1}{2^{\gamma/2} \Gamma(\gamma/2)} (\chi^2)^{\gamma/2 - 1} e^{-\chi^2/2}.$$

Let β be a beta random Variable with parameters α and β . Then the density of β is given by

$$f(\beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \Gamma(\beta)} \beta^{\alpha - 1} (1 - \beta)^{\beta - 1}.$$

Let U be a uniform random Variable on the interval $[0, 1]$. Then the density of U is given by

$$f(u) = 1, \quad 0 \leq u \leq 1.$$

Let A_1 be a standard normal random Variable. Then the density of A_1 is given by

$$f(a_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{a_1^2}{2}}.$$

Let p and n be independent standard normal random Variables. Then the joint density of (p, n) is given by

$$f(p, n) = \frac{1}{2\pi} e^{-\frac{p^2 + n^2}{2}}.$$

Let η be a standard normal random Variable. Then the density of η is given by

$$f(\eta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\eta^2}{2}}.$$

Let W be a standard normal random Variable. Then the density of W is given by

$$f(w) = \frac{1}{\sqrt{2\pi}} e^{-\frac{w^2}{2}}.$$

$$\mathbf{WA} = \begin{bmatrix} -0.003 & 0.000 & 0.001 & -0.001 & \mathbf{1.000} & 0.000 & -0.002 & -0.003 \\ -0.002 & 0.000 & -0.002 & 0.001 & -0.002 & 0.000 & \mathbf{-1.000} & -0.001 \\ 0.000 & 0.002 & 0.000 & \mathbf{-1.000} & -0.001 & -0.004 & -0.001 & -0.002 \\ 0.001 & -0.003 & \mathbf{-1.000} & 0.000 & 0.001 & 0.000 & 0.002 & 0.002 \\ -0.003 & \mathbf{-1.000} & 0.003 & -0.002 & 0.000 & 0.001 & 0.000 & 0.001 \\ -0.002 & -0.001 & -0.002 & 0.002 & -0.003 & 0.002 & 0.001 & \mathbf{-1.000} \\ -0.001 & 0.001 & 0.000 & -0.004 & 0.000 & \mathbf{1.000} & 0.000 & 0.002 \\ \mathbf{1.000} & -0.003 & 0.001 & 0.000 & 0.003 & 0.001 & -0.002 & -0.003 \end{bmatrix} \quad (11)$$

$$\mathbf{WA} = \begin{bmatrix} 0.021 & -0.002 & -\mathbf{1.449} & 0.029 & -0.014 & 0.038 & -0.017 & -0.053 \\ 0.009 & -0.057 & -0.039 & -0.033 & 0.053 & \mathbf{1.430} & -0.008 & -0.022 \\ \mathbf{1.450} & -0.040 & -0.014 & -0.010 & 0.033 & 0.052 & 0.009 & -0.076 \\ 0.050 & -0.035 & -0.047 & 0.045 & -0.014 & 0.025 & -0.008 & -\mathbf{1.499} \\ -0.037 & -\mathbf{1.452} & 0.031 & -0.040 & -0.048 & 0.037 & -0.038 & -0.023 \\ -0.031 & -0.046 & -0.031 & 0.007 & -\mathbf{1.444} & -0.047 & -0.005 & -0.023 \\ -0.014 & 0.015 & 0.037 & 0.022 & 0.037 & -0.016 & \mathbf{1.443} & 0.032 \\ -0.013 & 0.056 & -0.006 & \mathbf{1.404} & 0.014 & -0.018 & 0.057 & -0.013 \end{bmatrix} \quad (12)$$

3.3 On Separating Audio Sources

http://
www-bcl.cs.may.ie/~bap/demos.html

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