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[1] ...
[2] ... A ... 351-0198,
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Abstract. ... (A) ... A ... A ... A ... A ...

I...

... A ... A ... A ... $\mathbf{x}$ ... $\mathbf{A}\mathbf{s}$ ... $\mathbf{A} \in \mathbb{R}^{m \times n}$ ... $\mathbf{s} \in \mathbb{R}^{n}$ ... $\mathbf{x} \in \mathbb{R}^{m}$ ... n ... m ... m ... A ... $\mathbf{W} \in \mathbb{R}^{n \times m}$ ... $\mathbf{x}$

$$\mathbf{y} \quad \mathbf{W}\mathbf{x} \quad \mathbf{W}\,\mathbf{A}\mathbf{s} \quad \mathbf{W}\mathbf{A}\,\mathbf{s}$$

... $\mathbf{y} \in \mathbb{R}^{n}$ ... $\mathbf{s}$ ... $\mathbf{y}$ ...

J. Rosca et al. (Eds.): ICA 2006, LNCS 3889, pp. 838–845, 2006.

$$q(\mathbf{y}) = \prod_{i=1}^{n} q(y_i),$$

[illegible]

$$D(\mathbf{W}) = -H(\mathbf{y}) - \sum_{i=1}^{n} \int p(y_i|\mathbf{W}) \log p_i(y_i)\, dy_i,$$

[illegible]

globally

locally

a a a

$$J\ \mathbf{R}\quad \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^{4}\nu_{j}^{s}k_{i}^{m},$$

$\mathbf{R}$ r_{ij} $n\times n$ **WA** $\mathbf{R}$ **W** ν_j^s s_j k_i^m ν_i^m **K**

$$\mathbf{K}\quad k_{ij}\ {}_{n\times n},\quad k_{ij}\quad \nu_{j}^{s}k_{i}^{m}.$$

$$J\ \mathbf{R} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^{4}\nu_{j}^{s}k_{i}^{m} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^{4}k_{ij}.$$

$k_1^m \geq \cdots \geq k_p^m >$ $> k_{p+1}^m \geq \cdots \geq k_n^m$ $\nu_1^s \geq \cdots \geq \nu_p^s >$ $> \nu_{p+1}^s \geq \cdots \geq \nu_n^s$

R n J **R** J **R** **A** J **R** **A**

Lemma 1. *Suppose that* $F\ \mathbf{x}$ *(*$\mathbf{x} \in \mathbb{R}^m$ *is a twice differentiable scalar function under the following constraints:*

$$C_i\ \mathbf{x} \quad , \quad i \quad , \ , \cdots , k.$$

Construct a Lagrange function with a Lagrange multiplier set $\boldsymbol{\lambda}$ $\{\lambda_1, \lambda_2, \cdots, \lambda_k\}$*, i.e.,* $L\ \mathbf{x}, \boldsymbol{\lambda}$ $F\ \mathbf{x}$ $\sum_{i=1}^{k} \lambda_i C_i\ \mathbf{x}$ *, and assume that* $\mathbf{x}^*, \boldsymbol{\lambda}^*$ *is a solution of the system of the equalities that all the derivatives of* $L\ \mathbf{x}, \boldsymbol{\lambda}$ *with respect to the variables of* $\mathbf{x}$ *and the Lagrange multipliers* λ_i *are equal to zeros. It is also assumed that these* $\nabla C_i\ \mathbf{x}^*$ *are linearly independent. If for any nonzero vector* $\mathbf{q}$ / *under the constraints* $\mathbf{q}^T \nabla_{\mathbf{x}} C_i\ \mathbf{x}^*$ *for* i , , $\cdots , k$*, we have*

$$\mathbf{q}^T \nabla_{\mathbf{x}}^2 L\ \mathbf{x}^*, \boldsymbol{\lambda}^*\ \mathbf{q} < \quad \text{or} > \quad ,$$

then $\mathbf{x}^*$ *is a local maximum (or local minimum) of* $F\ \mathbf{x}$ *under the constraints.*

a

J **R** **R**

s m m s s n m m x s n m m x m , J **R** n x m x **K** $\mathbf{R}\mathbf{R}^T$ **I**

s n s s m m s m s **λ** $\{\lambda_{ij} \quad i \le j\}$ s n

$$L \; \mathbf{R}, \boldsymbol{\lambda} \quad \sum_{i=1}^{n}\sum_{j=1}^{n} r_{ij}^4 k_{ij} \quad \sum_{i=1}^{n}\sum_{j=i}^{n} \lambda_{ij}\Big(\sum_{l=1}^{n} r_{li} r_{lj} - \delta_{ij}\Big),$$

n δ_{ij} s n n

$$\frac{\partial L(\mathbf{R}, \boldsymbol{\lambda})}{\partial r} = 4k \;\; r^3 + \sum_{=1}^{1} r \;\; \lambda \;\; + 2\lambda \;\; r \;\; + \sum_{= \; +1} r \;\; \lambda \;\; ; \tag{10}$$

$$\frac{\partial L(\mathbf{R}, \boldsymbol{\lambda})}{\partial \lambda} = \sum_{=1} r \;\; r \;\; - \delta \;\; . \tag{11}$$

λ m x **U** $u_{ij} \;\; n \times n$ s

$$u \;\; = \begin{cases} \lambda \\ \\ \end{cases}$$

Theorem 1. *If* $\mathbf{R}^*$ *is a permutation matrix up to sign indeterminacy and* $k_{ij} >$ *at all the positions where* $|r^*_{ij}|$ *, it corresponds to a local maximum of the objective function* J $\mathbf{R}$.

Proof: $n \times n$ m x $\mathbf{R}$ $n^2 \times$

$$vec\,\mathbf{R} \quad r_{11}, r_{21}, \cdots, r_{n1}, r_{12}, r_{22}, \cdots, r_{n2}, \cdots, r_{1n}, r_{2n}, \cdots, r_{nn}{}^{T} \in \mathbb{R}^{n^2}.$$

s m s s $n^2 \times$ $\mathbf{q}$

$$\mathbf{q} \quad q_{11}, q_{21}, \cdots, q_{n1}, q_{12}, q_{22}, \cdots, q_{n2}, \cdots, q_{1n}, q_{2n}, \cdots, q_{nn}{}^{T} \in \mathbb{R}^{n^2}.$$

n s

$$\frac{\partial^2 L\ \mathbf{R}, \boldsymbol{\lambda}}{\partial r_{ij} \partial r_{i'j'}} \quad \delta_{(i,j),(i',j')} \quad k_{ij} r_{ij}^2 \quad u_{jj} ,$$

n $\delta_{(i,j),(i',j')}$ s n s n n s i', j' i, j m i i' j j' n s s m n

$$\mathbf{U} \quad - \ \mathbf{R}^T \mathbf{B}.$$

n $\mathbf{R}$ $\mathbf{R}^*$ s m m x s m $\mathbf{U}^*$ ss n $\boldsymbol{\lambda}^*$ m x n n $k_{ij} >$ *every* $|r^*_{ij}|$ s n u^*_{jj}

Theorem 2. *If* $\mathbf{R}^*$ *is a permutation matrix up to sign indeterminacy and* $k_{ij} <$ *at all the positions where* $|r^*_{ij}|$ *, it corresponds to a local minimum of the objective function* $J(\mathbf{R})$.

Remark 2.

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