A Fixed-Point EM Algorithm for Straight Line Detection

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Abstract. Straight line detection is a basic technique in image processing and pattern recognition. It has been investigated from different aspects, but is still very challenging in practical applications. In this paper, based on the finite mixture model and under the EM framework, we maximize the Q-function by differentiation and construct a fixed-point EM algorithm for straight line detection. It is demonstrated by the experiments that this proposed algorithm can effectively detect the straight lines from a digital image or dataset.

Keywords: Straight line detection, Expectation Maximization (EM), Fixed-Point iteration.

1 Introduction

S aigh li e de ec i i a ba ic ech i e i i age ce i g a d a e ec g- i i . I fac , i i a ce f l ca i g he aigh li e f a digi al i age 2-di e i al da a e a d he e a e a va ie , f lea i g alg i h f aigh li e de ec i . I he li e a e, he H gh a f (HT) [1] a d i e xe i [2] a e i a l f aigh li e de ec i . Ge e all , he H gh a f ffe f heav c a i al c . I de alleviate hi ea e , Ra d H gh T a f (RHT) [3] a d he c ai ed H gh T a f [4] e e f he e abli hed. F he he a ec , he e have al e abli hed a lea i g alg i h f aigh li e c ve de ec i (e.g., [5]-[8]).

The l cal i ci al c e a al i (PCA) alg i h [9]-[12], a a e xe i f PCA [13], i f e ed f aigh li e de ec i . I i le e he lea ea a e e ec c i (LMSER) i ci le [14] a d de ec he aigh li e via i i i i g he f ll i g c f c i [11]:

$$E = \sum_{k=1}^{K} E_k = \sum_{k=1}^{K} \sum_{x_t \in \mathcal{L}_k} d^2(x_t, \mathcal{L}_k) \qquad (1)$$

he e x_t i he t- h a le i be g i g he li e $\mathcal{L}_k$ a d $d(x_t, \mathcal{L}_k)$ de e he E clidea di a ce f he da a i x_t he li e $\mathcal{L}_k$. Ac all , he li e $\mathcal{L}_k$ ca be c ide ed a a ecial b e f i . O he he ha d, he li e $\mathcal{L}_k$

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ca be al egarded a d e . A all che i i he he k- h aigh li e i he fi i ci al c f he d e $\mathcal{L}_k$. The ef e, he l i f $\mathcal{L}_k$ i j he a a f he l cal i ci al c e a al i e h d.

Each x_t i a ig ed a aigh a ifica i e be hi f c i give b

$$I(x_t, k) = \begin{cases} 1 & \text{if } k \quad d(x_t, \mathcal{L}_j),\ j = 1, \cdots, K \\ 0 & \text{he} \end{cases} \tag{2}$$

I fac , x_t bel g $\mathcal{L}_k$ if a d $k) = 1$. I ea ha he di a ce f x_t $\mathcal{L}_k$ i he alle e ca da e $\mathcal{L}_k$ b he f ll i g le: $\mathcal{L}_k$ i he fi i ci al c e b e $\mathcal{L}_k$. Rece l , he RPCL al g i h [15] a c bi ed i h PCA al g i h f aigh li e de ec i [6].

I hi a e , ili e he fi i de l f aigh li e de ec i . I i ea l ve he a i f he l g-li elih d f c i di- ec l . S , e e he E ec a M a i (EM) [16] a d a al e he Q-f c i . B he diffe e ia i he i , e c c a fixed- i EM al g i h f aigh li e de e . a ed b he e e i e ha he ed fixed- i EM a i h de ec he aigh li e f a da a e effec ivel .

The e f he a e i ga i e a f l We begi i d ce he fi i e i e del f aigh li e de e i 2. We he de ive a d e e fixed- i EM al g i h i Se 3 e i e al e l a e f he de a ed i Sec i 4. Fi all , b ie i i ade i Sec i 5.

2 The Finite Mixture M del raight Line Detection

He e, i i a ed ha he f li e i lea i g de l i e al he be f ac al aigh i he i age da a e . F i lici , e l f c he 2-d e i a i , b he de iva i a d a al i ca be ea il ge e ali ed he i h highe di e i . Le he da a e be de ed b $\mathcal{S} = \{x_t$

where $(x_t - m_k, \ell_k)$ denotes the inner product of $x_t - m_k$ and ℓ_k.

In this situation, we can establish the following finite mixture model:

$$q(x|\Theta_K) = \sum_{j=1}^{K} \pi_j q(x|\ell_j, m_j, \sigma_j), \tag{7}$$

where

$$q(x|\ell_j, m_j, \sigma_j) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\{-\frac{d^2(x_t, \ell_j, m_j)}{2\sigma_j^2}\}, \tag{8}$$

$$\|\ell_j\|^2 = {\ell_{j1}}^2 + {\ell_{j2}}^2 = 1, \quad j = 1, \ldots, K, \tag{9}$$

$$\sum_{j=1}^{K} \pi_j = 1. \tag{10}$$

In this mixture model, π_i represents the mixing proportion, $d^2(x_t, \ell_k, m_k)$ denotes the distance between the point x_t and the line with parameters ℓ_k and m_k. σ_i can be considered as the noise level of the dataset. That is, when σ_i is large, the noise level is high.

As it is a finite mixture model, we are difficult to find the maximum of its log-likelihood function directly. So, we resort to the EM algorithm [16]. Under the EM framework, we introduce a missing variable j and construct the Q-function:

$$Q(\Theta_K^h, \Theta_K^{h+1}) = \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{K} p(j|x_t, \Theta_K^h) \ln q(x_t|j, \Theta_K^{h+1}) \tag{11}$$

$$= \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{K} \frac{\pi_j^h q(x_t|\ell_j^h, m_j^h, \sigma_j^h)}{\sum_{i=1}^{K} \pi_i^h q(x_t|\ell_j^h, m_j^h, \sigma_j^h)} \ln[\pi_j^{h+1} q(x_t|\ell_j^{h+1}, m_j^{h+1}, \sigma_j^{h+1})]$$

$$= \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{K} p_j(t) \ln[\pi_j^{h+1} q(x_t|\ell_j^{h+1}, m_j^{h+1}, \sigma_j^{h+1})] \tag{12}$$

where $p_j(t) = \frac{\pi_j^h q(x_t|\ell_j^h, m_j^h, \sigma_j^h)}{\sum_{i=1}^{K} \pi_i^h q(x_t|\ell_j^h, m_j^h, \sigma_j^h)}$. For simplicity, the Q-function is denoted by

$$Q = \frac{1}{N} \sum_{t=1}^{N} \sum_{j=1}^{K} p_j(t) \ln[\pi_j q(x_t|\ell_j, m_j, \sigma_j)]. \tag{13}$$

3 Proposed Fixed-Point EM Algorithm

In the EM algorithm, it is key to solve the maximum of the Q-function. We now analyze the Q-function and try to establish a fixed-point learning algorithm to solve the maximum of Q-function.

Since $\sum_{j=1}^{K} \pi_j = 1$ and ${\ell_{j1}}^2 + {\ell_{j2}}^2 = 1$ for any j, we introduce the Lagrange multipliers $\beta, \lambda_j (j = 1, \ldots, K)$ and the Lagrange function

$$L(\Theta_K, \beta, \lambda_1, \ldots, \lambda_K) = Q + \beta(1 - \sum_{j=1}^{K} \pi_j) + \sum_{j=1}^{K} \lambda_j (1 - \ell_{j1}^2 - \ell_{j2}^2). \tag{14}$$

B9 diffe e ia i , e have he f ll i g de iva ive :

$$\frac{\partial L}{\partial \pi_j} = \frac{1}{N}\sum_{j=1}^{K}\frac{1}{\pi_j}p_j(t) - \beta, \tag{15}$$

$$\frac{\partial L}{\partial \beta} = 1 - \sum_{j=1}^{K}\pi_j, \tag{16}$$

$$\frac{\partial L}{\partial \lambda_j} = 1 - l_{j1}^2 - l_{j2}^2, \tag{17}$$

$$\frac{\partial L}{\partial \ell_{j1}} = \frac{1}{N}\sum_{j=1}^{K}p_j(t)\frac{1}{\sigma_j^2}\{-(x_{t1}-m_{j1})[(x_{t1}-m_{j1})\ell_{j1} + (x_{t2}-m_{j2})\ell_{j2}]\}, \tag{18}$$

$$\frac{\partial L}{\partial m_{j1}} = \frac{1}{N}\sum_{j=1}^{K}p_j(t)\frac{1}{\sigma_j^2}\{-(x_{t1}-m_{j1}) + \ell_{j1}(x_t - m_j, \ell_j)\}, \tag{19}$$

$$\frac{\partial L}{\partial \sigma_j} = \frac{1}{N}\sum_{j=1}^{K}p_j(t)\{\frac{1}{\sigma_j} + \frac{1}{\sigma_j^3}d^2(x_t, \ell_j, m_j)\}. \tag{20}$$

B9 le i g he e de iva ive give b9 E . (15)-(20) be 0, e have

$$\beta = \frac{1}{N}\sum_{j=1}^{K}\sum_{t=1}^{N}p_j(t), \tag{21}$$

a d f he b ai he f ll i g fi xed- i lea i g al g i h :

$$m_j^{h+1} = \frac{\sum_t p_j(t)x_t}{\sum_t p_j(t)}, \tag{22}$$

$$\pi_j^{h+1} = \frac{1}{N}\sum_t p_j(t), \tag{23}$$

$$(\sigma_j^{h+1})^2 = \frac{\sum_t p_j(t)d^2(x_t, \ell_k, m_k)}{\sum_t p_j(t)}, \tag{24}$$

a d ℓ_j i he eige vec f $\Sigma_j = \sum_{j=1}^{N} p_j(t)(x_t - m_j)(x_t - m_j)^T$ c e di g he la ge eige val e.

Ba ed he ab ve fi xed- i lea i g al g i h , e ca e abli h he fi xed- i EM al g i h hich c i f he f ll i g h ee e :

(i) I i iali a i f he a a e .

(ii) U da e m_j, π_j, σ_j^2 b9 E . (22)-(24). U da e ℓ_j b9 he eige vec f $\Sigma_j = \sum_{j=1}^{N} p_j(t)(x_t - m_j)(x_t - m_j)^T$ c e di g he la ge eige val e.

(iii) Re ea S e (ii) il he val e f a a e a e cha ged.

4 Experiments Results

I hi ec i , eve al i la i e x e i e a e ca ied de a e he e f a ce f he fixed- i EM alg i h f aigh li e de ec i . We c ide he bi a i age da a e f f aigh li e i h h ee i d f i e . The e a a e f he fi i e i x e de f he h ee da a e a e li ed i Table 1. Obvi l , he i e level i $\mathcal{S}_2$ a d $\mathcal{S}_3$ a e ch highe ha ha f $\mathcal{S}_1$.

I e x e i e , e e he be f aigh li e be he e be f aigh li e , i.e., $K = 4$. We i le e he fixed- i EM alg i h each da a e , i h he a a e e bei g i i iali ed b he a d H gh a f [3]. The alg i h if $|Q(\Theta_K^{new}) - Q(\Theta_K^{old})| < 10^{-6}$. The e l f he aigh li e de ec i a ell a he b ai ed Q-f c i a e h i Fig. 1-3, e ec ivel . The lea ed a a e e f he fi i e i x e de each e x e i e a e li ed i Table. 2.

Table 1. The true parameters of the finite mixture models for the datasets $\mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{S}_3$, respectively

Sample set	π_i	ℓ_i	m_i	σ_i
$\mathcal{S}_1$	0.25	(-0.7071,0.7071)	(1,1)	0.01
	0.25	(-0.7071,0.7071)	(-1,1)	0.01
	0.25	(-0.7071,0.7071)	(-1,-1)	0.01
	0.25	(0.7071,0.7071)	(1,-1)	0.01
$\mathcal{S}_2$	0.25	(-0.7071,0.7071)	(1,1)	0.2
	0.25	(-0.7071,0.7071)	(-1,1)	0.2
	0.25	(-0.7071,0.7071)	(-1,-1	0.2
	0.25	(0.7071,0.7071)	(1,-1)	0.2
$\mathcal{S}_3$	0.25	(-0.7071,0.7071	(1,1)	0.3
	0.25	(-0.7071,0.7071	(-1,1)	0.3
	0.25	(-0.7071,0.7071	(-1,-1	0.3
	0.25	(0.7071,0.7071)	(1,-1)	0.3

I ca be b e ved f he fig e i Fig. 1 ha he Q-f c i i c ea d i g he i e a i a d fi all each e i a i . Mea hile, he aigh li e a e acc a el ca ed i each ca e. We ca al b e ve ha he Q-f c i i c ea ha l a he begi i g f he i e a i . The ea a be ha he a a e a e i i iali ed b he a d H gh a f , bei g l e gh e i a e f he a a e . A he i i iali a i f he a a e be c e be e , he c ve f he Q-f c i ill be e h.

F he e e x e i e al e l , e a e e ha ed fixed- i EM alg i h ca effec ivel de ec he aigh li e i all he h ee da a e i h diffe e i e level . M e ve , i i h i Fig. 3-(c) ha he fixed- i EM alg i h al e f ell he gl i i a i .

I i i ha he be f aigh li e i he lea i g i x e de h ld be i advance. B hi i f a i a be available i

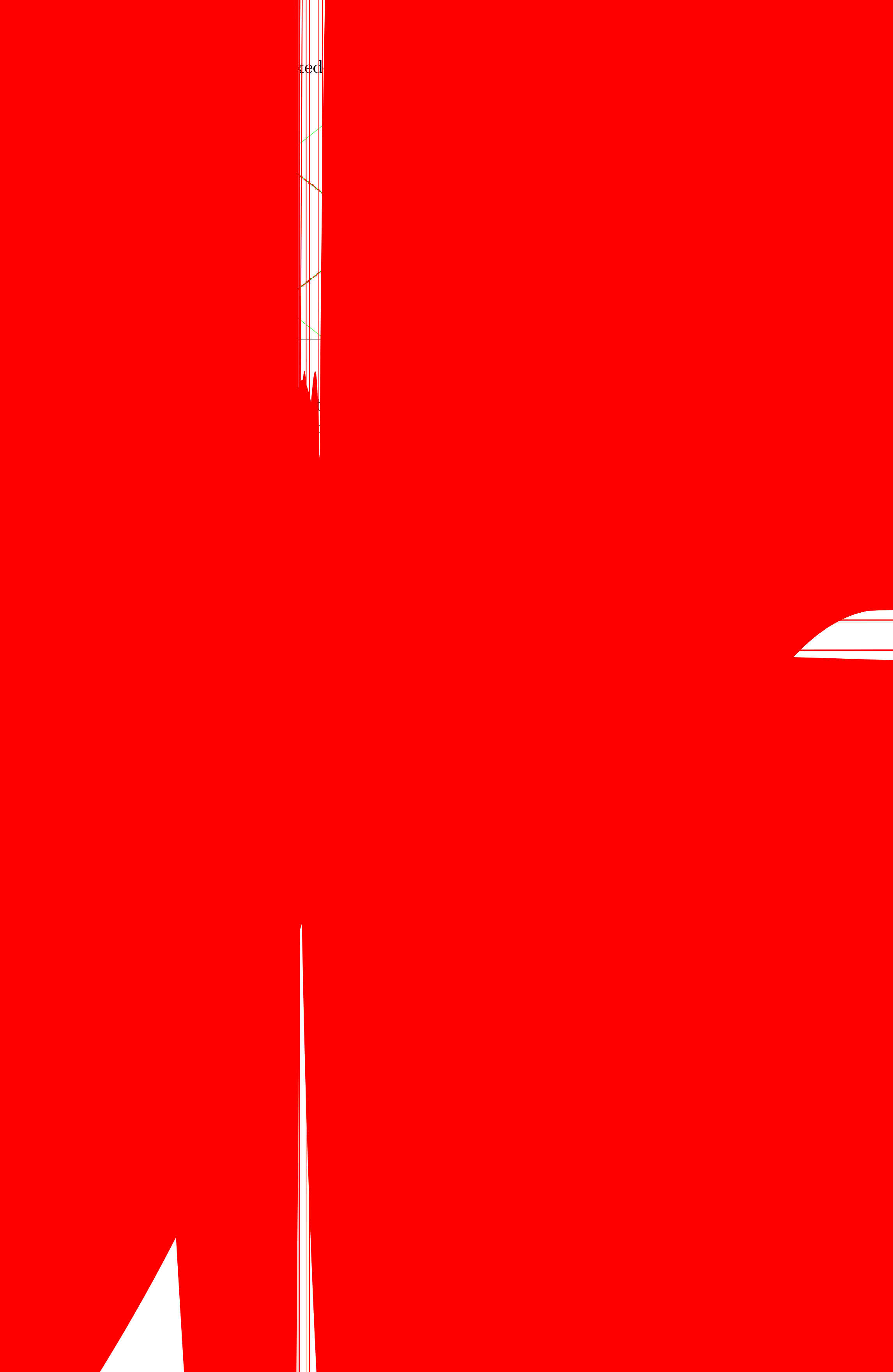

Table 2. The learned parameters of the finite mixture models on the three datasets $\mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{S}_3$, respectively

Sample set	π_i	ℓ_i	m_i	σ_i
$\mathcal{S}_1$	0.2498	(-0.7084,0.7058)	(0.9715,1.0290)	0.0103
	0.2489	(-0.7084,0.7058)	(-0.9695,1.0302)	0.0083
	0.2502	(-0.7077,0.7065)	(-0.9516,-1.0491)	0.0089
	0.2511	(0.7067,0.7075)	(1.0120,-0.9871)	0.0097
$\mathcal{S}_2$	0.2617	(0.7101,-0.7041)	(1.0192,0.9542)	0.1726
	0.2421	(0.7119, 0.7023)	(-1.0138,0.9835)	0.1812
	0.2380	(0.7210,-0.6930)	(-0.9720,-1.0285)	0.1760
	0.2582	(-0.7157,-0.6984)	(0.9778,-0.9707)	0.2138
$\mathcal{S}_3$	0.2386	(-0.7908,0.6120)	(0.8080,1.0306)	0.2695
	0.2557	(-0.7459,-0.6661)	(-0.8380,1.0016)	0.2826
	0.2335	(0.7296,-0.6839)	(-0.8854,-0.9882)	0.2867
	0.2722	(-0.7323,-0.6810)	(0.8837,-0.9461)	0.3386

practical applications. In order to overcome this weakness, we can introduce the Bayesian Ying-Yang (BYY) harmony learning system [17]-[18] and the entropy penalized automated model selection mechanism [19] into the fixed-point learning algorithm.

5 Conclusions

We have investigated the straight line detection problem from the finite mixture modeling. Since it is difficult to solve the maximum of the log-likelihood function, we use the EM algorithm and analyze the Q-function. By differentiation, we derive a fixed-point learning procedure for maximizing the Q-function and thus construct a fixed-point EM algorithm for straight line detection. It is demonstrated by the experiments that the proposed fixed-point EM algorithm can effectively locate the straight lines in a dataset.

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References

1. Ballard, D.: Generalizing the Hough Transform to Detect Arbitrary Shapes. Pattern Recognition 13(2), 111–122 (1981)
2. Wolfson, H.: Generalizing the Generalized Hough Transform. Pattern Recognition Letters 12(9), 565–573 (1991)
3. Xu, L., Oja, E., Kultanen, P.: A New Curve Detection Method - Randomized Hough Transform (RHT). Pattern Recognition Letters 11(5), 331–338 (1990)

4. Olson, C.F.: Constrained Hough Transforms for Curve Detection. Computer Vision and Image Understanding 73, 329–345 (1998)
5. Liu, Z., Qiao, H., Xu, L.: Multisets Mixture Learning-Based Ellipse Detection. Pattern Recognition 39(4), 731–735 (2006)
6. Liu, Z., Chiu, K., Xu, L.: Strip Line Detection and Thinning by RPCL-Based Local PCA. Pattern Recognition Letters 24(14), 2335–2344 (2003)
7. Lu, Z.W., Cheng, Q.S., Ma, J.W.: A Gradient BYY Harmony Learning Algorithm on Mixture of Experts for Curve Detection. In: Gallagher, M., Hogan, J.P., Maire, F. (eds.) IDEAL 2005. LNCS, vol. 3578, pp. 250–257. Springer, Heidelberg (2005)
8. Chen, G., Li, L., Ma, J.: A Gradient BYY Harmony Learning Algorithm for Straight Line Detection. In: Sun, F., Zhang, J., Tan, Y., Cao, J., Yu, W. (eds.) ISNN 2008, Part I. LNCS, vol. 5263, pp. 618–626. Springer, Heidelberg (2008)
9. Xu, L.: Multisets Modeling Learning: an Unified Theory for Supervised and Unsupervised Learning. In: IEEE International Conference on Neural Networks, 1994. IEEE World Congress on Computational Intelligence, vol. 1, pp. 315–320 (1994)
10. Kambhatla, N., Leen, T.: Dimension Reduction by Local Principal Component Analysis. Neural Computation 9(7), 1493–1516 (1997)
11. Xu, L.: Vector Quantization by Local and Hierarchical LMSER. In: Proc. of 1995 Intl. Conf. on Artificial Neural Networks, vol. 2, pp. 575–579 (1995)
12. Xu, L.: An Overview on Unsupervised Learning from Data Mining Perspective. In: Allinson, N., Yin, H., Allinson, L., Slack, J. (eds.) Advances in Self-Organising Maps, pp. 181–209 (2001)
13. Jolliffe, I.T.: Principal Component Analysis, 2nd edn. Springer, Heidelberg (2002)
14. Xu, L.: Least Mean-Square Error Reconstruction Principle for Self-Organizing Neural-Nets. Neural Networks 6(5), 627–648 (1993)
15. Xu, L., Krzyzak, A., Oja, E.: Rival Penalized Competitive Learning for Clustering Analysis, RBF Net, and Curve Detection. IEEE Transactions on Neural Networks 4(4), 636–649 (1993)
16. Dempster, A., Laird, N., Rubin, D.: Maximum Likelihood from Incomplete Data via EM Algorithm. Journal of the Royal Statistical Society Series B-Methodological 39(1), 1–38 (1977)
17. Ma, J.W., He, X.F.: A Fast Fixed-Point BYY Harmony Learning Algorithm on Gaussian Mixture with Automated Model Selection. Pattern Recognition Letters 29(6), 701–711 (2008)
18. Ma, J.W., Liu, J.F.: The BYY Annealing Learning Algorithm for Gaussian Mixture with Automated Model Selection. Pattern Recognition 40(7), 2029–2037 (2007)
19. Ma, J.W., Wang, T.J.: Entropy Penalized Automated Model Selection on Gaussian Mixture. International Journal of Pattern Recognition and Artificial Intelligence 18(8), 1501–1512 (2004)