modeling with a favorite feature that model selection can be made automatically during parameter learning. That is, they can learn the correct number of actual Gaussians in a sample data set automatically. From the data space of a binary image, all the black pixels or points are regard as the samples or sample points generated from the image and the distance from a sample point to the straight line it is along with, is subject to some Gaussian distribution or function, since there always exists some noise. Thus, the straight lines can be represented through some Gaussians and their detection in a binary image is equivalent to the Gaussian mixture modeling of both automated model selection and parameter learning, which can be certainly solved by this kind of BYY harmony learning on Gaussian mixture. On the other hand, according to the BYY harmony learning on the mixture of experts, a gradient learning algorithm was already proposed in [14] for the straight line or ellipse detection, but it was not applicable for the general case.

In this paper, with straight lines being implicitly represented by the Gaussians of the distances from samples to them, authors propose a new gradient BYY harmony learning algorithm for straight line detection based on the gradient BYY harmony learning rule established in [9]. It is demonstrated well by the experiments that this gradient BYY harmony learning algorithm approach can efficiently determine the number of straight lines and locate these straight lines accurately in an image.

In the sequel, authors introduce the BYY learning system and the harmony function and propose the gradient BYY harmony learning for straight line detection in Section 2. In Section 3, several experiments on both the simulation and real-world images are conducted to demonstrate the efficiency of our proposed algorithm. Finally, authors will conclude briefly in Section 4.

2.1 BYY Learning System and the Harmony Function

A BYY system describes each observation $x \in \quad \subset \Re^n$ and its corresponding inner representation $y \in \quad \subset \Re^m$ via the two types of Bayesian decomposition of the joint density: $p(x,y) = p(x)p(y|x)$ and $q(x,y) = q(y)q(x|y)$, which are called Yang machine and Ying machine, respectively. Given a data set $D_x = \{x_t\}_{t=1}^N$ from the Yang or observable space, the goal of harmony learning on a BYY learning system is to extract the hidden probabilistic structure of x with the help of y from specifying all aspects of $p(y|x)$, $p(x)$, $q(x|y)$ and $q(y)$ via a harmony learning principle implemented by maximizing the functional

$$H(p||q) = \int p(y|x)p(x)ln[q(x|y)q(y)]dxdy, \qquad (1)$$

which is essentially equivalent to minimizing the Kullback-Leibler divergence between the Yang and Ying machines, i.e., $p(x,y)$ and $q(x,y)$, because

$$KL(p||q) = \int p(y|x)p(x)ln\frac{p(y|x)p(x)}{q(x|y)q(y)}dxdy = -H(p||q) - H(p),$$

where $H(p)$ is the entropy of $p(x, y)$ and invariant to $q(x, y)$.

If both $p(y|x)$ and $q(x|y)$ are parametric, i.e. from a family of probability densities with parameter θ, the BYY learning system is said to have a BI-directional Architecture (BI-Architecture for short). For the Gaussian mixture model with a given sample set $D_x = \{x_t\}_{t=1}^N$, we can utilize the following specific BI-architecture of the BYY learning system. The inner representation y is discrete in $\ = \{1, 2, \cdots, k\}$ (i.e., with $m = 1$), and the observation x comes from a Gaussian mixture distribution. On the Ying space, we let $q(y = j) = \alpha_j \geq 0$ with $\sum_{j=1}^k \alpha_j = 1$. On the Yang space, we suppose that $p(x)$ is a blind Gaussian mixture distribution, with a set of sample data D_x being generated from it. Moreover, in the Ying path, we let each $q(x|y = j) = q(x|\theta_j)$ be a Gaussian probability density function (pdf) given by

$$q(x|\theta_j) = q(x|m_j, \Sigma_j) = \frac{1}{(2\pi)^{\frac{}{2}}|\Sigma_j|^{\frac{1}{2}}}e^{-\frac{1}{2}(x-m\ )\ \ \Sigma\ \ ^{1}(x-m\ )}, \tag{2}$$

where m_j is the mean vector and Σ_j is the covariance matrix which are assumed positive definite. On the other hand, the Yang path is constructed under the Bayesian principle by the following parametric form:

$$p(y = j|x) = \frac{\alpha_j q(x|\theta_j)}{q(x|\Theta_k)}, \qquad q(x|\Theta_k) = \sum_{j=1}^k \alpha_j q(x|\theta_j), \tag{3}$$

where $\Theta_k = \{\alpha_j, \theta_j\}_{j=1}^k$ and $q(x|\Theta_k)$ is just a Gaussian mixture that will approximate the true Gaussian mixture $p(x)$ hidden in the sample data D_x via the harmony learning on the BYY learning system.

With all these component densities into Eq.(1), we have

$$H(p||q) = E_{p(x)}[\sum_{j=1}^k \frac{\alpha_j q(X|\theta_j)}{\sum_{i=1}^k \alpha_i q(X|\theta_i)} ln[\alpha_j q(X|\theta_j)]], \tag{4}$$

that is, it becomes the expectation of a random variable $\sum_{j=1}^k \frac{\alpha\ q(X|\theta\ )}{\sum\ _{=1}\alpha\ q(X|\theta\ )}$ $ln[\alpha_j q(X|\theta_j)$ where X is just the random variable (or vector) subject to $p(x)$. Based on the given sample data set D_x, we get an estimate of $H(p||q)$ as the following harmony function for Gaussian mixture with the parameter set Θ_k:

$$J(\Theta_k) = \frac{1}{N}\sum_{t=1}^N \sum_{j=1}^k \frac{\alpha_j q(x_t|\theta_j)}{\sum_{i=1}^k \alpha_i q(x_t|\theta_i)} ln[\alpha_j q(x_t|\theta_j)]. \tag{5}$$

2.2 The Gradient Learning Rule for Straight Line Detection

In order to maximize the above harmony function $J(\Theta_k)$, Ma, Wang and Xu proposed a general (batch-way) gradient learning rule [9] for Gaussian mixture.

Although some new learning algorithms (e.g., [10]-[13]) have been already proposed to improve it, we still use it in this paper for its convenience of the generalization to the case of straight line detection. Actually, in the Gaussian mixture model, if we set

$$\alpha_j = e^{\beta} \Big/ \sum_{i=1}^{k} e^{\beta} ,$$

and substitute it into the harmony function given in Eq.(5), by the derivatives of $J(\Theta_k)$ respect to all the parameters, we can easily construct the general gradient learning rule proposed in [9].

For straight line detection, we can use the following Gaussian functions to implicitly represent the straight lines in the image:

$$q(u|l) = q(x,y|l) = \exp\{-\frac{(w_l^T(x,y)^T - b_l)^2}{2\tau_l^2 w_l^T w_l}\}, \tag{6}$$

where $u = (x, y)$ denotes the pair of two coordinates of a pixel point in the binary image. The sample data set $\{u_t = (x_t, y_t)\}_{t=1}^{N}$ consists of all the black pixel points in the binary image. In each Gaussian function, there are two parameters w_l^T and b_l, from which we can get the equation of the straight line it represents: $w_l^T x = b_l$. Suppose that there are k straight lines in the image or k Gaussian functions in our mixture model. Then, we can replace all these components in the general gradient learning rule [9] with these $q(u|l)$ and obtain the following new gradient learning rule for straight line detection::

$$\Delta w_l = \eta \frac{\alpha_l}{N} \sum_{t=1}^{N} h(l|u_t)U(l|u_t) \frac{-(w_l^T u_t - b_l)^2 w_l - (w_l^T u_t - b_l) w_l^T w_l u_t}{e^{r_l}(w_l^T w_l)^2}, \tag{7}$$

$$\Delta b_l = \eta \frac{\alpha_l}{N} \sum_{t=1}^{N} h(l|u_t)U(l|u_t) \frac{w_l^T u_t - b_l}{e^{2r_l}(w_l^T w_l)}, \tag{8}$$

$$\Delta r_l = \eta \frac{\alpha_l}{N} \sum_{t=1}^{N} h(l|u_t)U(l|u_t) \frac{-(w_l^T u_t - b_l)^2}{e^{2r_l}(w_l^T w_l)}, \tag{9}$$

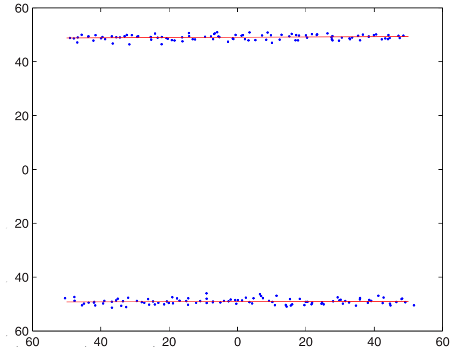
$$\Delta \beta_l = \eta \frac{\alpha_l}{N} \sum_{t=1}^{N} \sum_{j=1}^{k} h(j|u_t)U(j|u_t)(\delta_{jl} - \alpha_j), \tag{10}$$

where

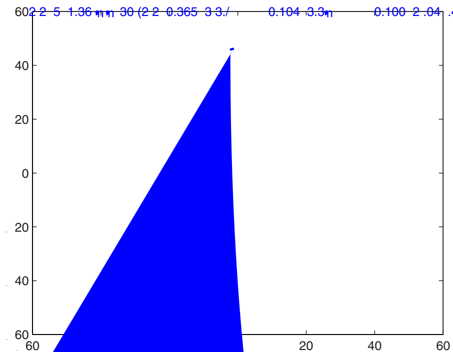
$$U(l|u_t) = 1 + \sum_{r=1}^{k} (\delta_{rl} - P(r|u_t)) \ln(\alpha_r q(u_t|r)), \tag{11}$$

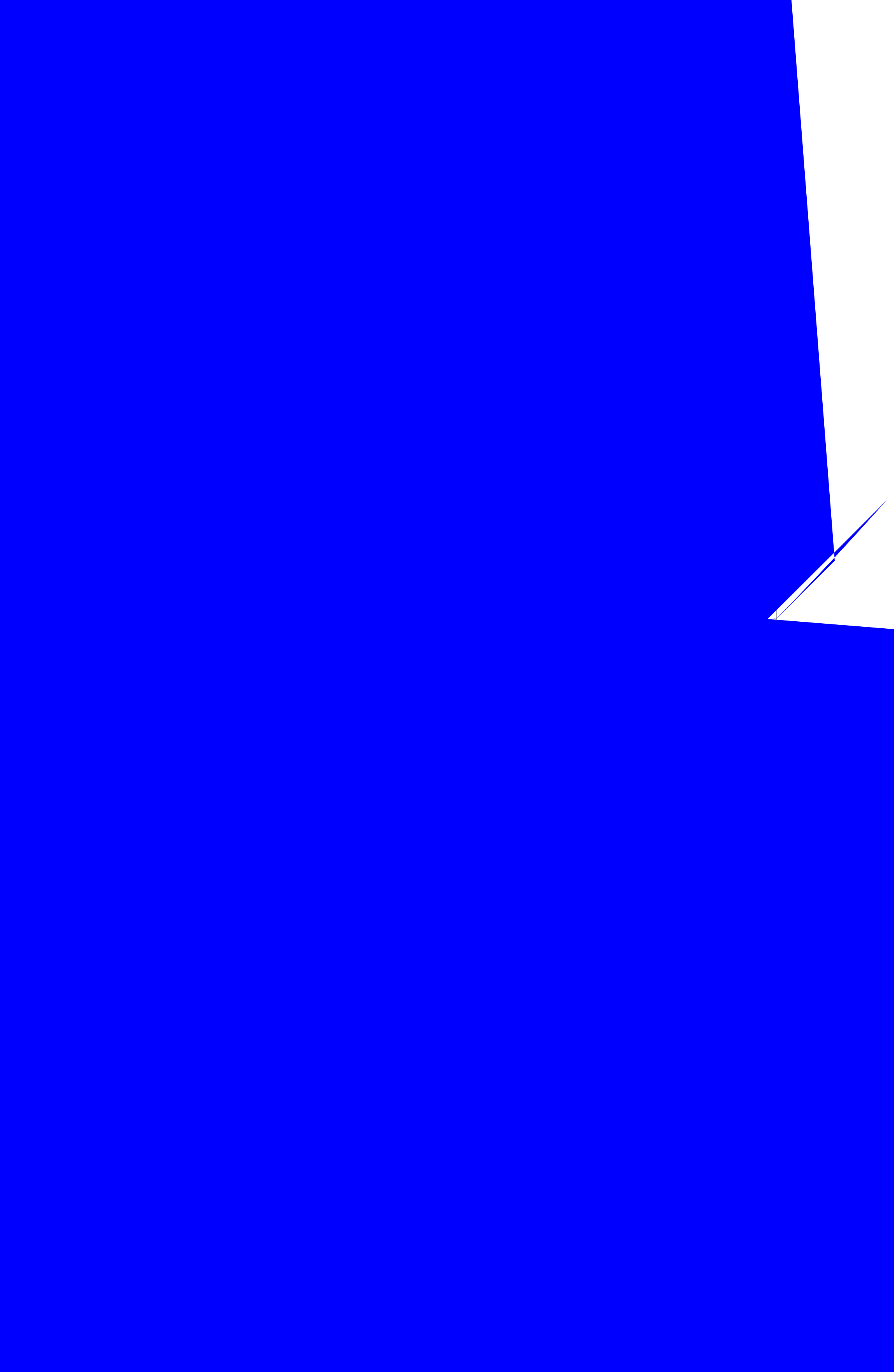
$$h(l|u_t) = q(u_t|l) / \sum_{r=1}^{k} \alpha_r q(u_t|r), P(r|u_t) = \alpha_r h(r|x_t). \tag{12}$$

where $\eta > 0$ is the learning rate, which will be selected from 0.01 to 0.1 in our experiments in the next section.



( ) rs S_1 k = 2





the determination of number of straight lines and the location of these straight lines. Moreover, the gradient BYY harmony learning algorithm is also applied to texture classification.

Using k^* to denote the true number of straight lines in the binary image, we implement the gradient BYY harmony algorithm on each set of binary image data with $k > k^*$, $\eta = 0.01$ and $\varepsilon = 0.05$. Moreover, the other parameters are initialized randomly within certain intervals. In all the experiments, the learning was stopped when $|J(\Theta_k^{new}) - J(\Theta_k^{old})| < 10^{-6}$.

We implement the gradient BYY harmony learning algorithm on the three sets of binary image data, which are shown in Fig.1(a),(b),(c), respectively. Actually, it can detect the actual straight lines in each binary image automatically and accurately. As shown in Fig.1(c), the algorithm is implemented on the third set S_3 of binary image data of four straight lines with $k = 6$. After the algorithm has converged, the mixing proportions of the two extra Gaussian functions or straight lines have been reduced to a very small number below 0.05 so that they can be discarded, while the other four lines are located accurately. Thus, the correct number of the straight lines in the image are detected automatically on this image data set. Moreover, a similar result of of the gradient BYY harmony learning has been made on the second image data set S_2 with $k = 6, k^* = 3$. As

We have proposed a new gradient BYY harmony learning algorithm for straight line detection. It is derived from the maximization of the harmony function on the mixture of Gaussian functions with the help of the general gradient BYY harmony learning rule. Several simulation experiments have demonstrated that the correct number of straight lines can be automatically detected on a binary image. Moreover, the gradient BYY harmony learning algorithm is successfully applied to the texture classification.

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