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**Abstract.** Let  $R$  be a local Artinian ring with residue field  $k$ . Let  $M$  be a finitely generated  $R$ -module. We study the structure of the socle of  $M$  and its relationship with the annihilator of  $M$ . We prove that the socle of  $M$  is isomorphic to the annihilator of  $M$  if and only if  $M$  is a direct sum of copies of the socle of  $R$ . We also study the structure of the socle of the direct sum of two modules.

**Keywords:** socle, annihilator, local Artinian ring, direct sum.

Let  $R$  be a local Artinian ring with residue field  $k$ . Let  $M$  be a finitely generated  $R$ -module. We study the structure of the socle of  $M$  and its relationship with the annihilator of  $M$ . We prove that the socle of  $M$  is isomorphic to the annihilator of  $M$  if and only if  $M$  is a direct sum of copies of the socle of  $R$ . We also study the structure of the socle of the direct sum of two modules.

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$$p(x|\Theta_k) = \sum_{i=1}^k \alpha_i p(x|\theta_i) = \sum_{i=1}^k \alpha_i p(x|\mu_i, \Sigma_i),$$

$$\sum_{i=1}^k \alpha_i \geq 1, \quad \alpha_i \geq 0, \quad p(x|\theta_i) = \frac{1}{\pi^{1/2} |\Sigma_i|^{1/2}} e^{-\frac{1}{2}(x-\mu_i)^T \Sigma_i^{-1}(x-\mu_i)},$$

$$\Theta_k = \{\theta_1, \dots, \theta_k, \alpha_1, \dots, \alpha_k\}$$

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$$N \dots \mathcal{X} \{x_t\}_{t=1}^N$$

$$p \mathcal{X} | \Theta_k \dots \prod_{t=1}^N p x_t | \Theta_k \dots \sum_{t=1}^N \dots \sum_{i=1}^k \alpha_i p x_t | \theta_i ,$$

$$b \dots \Theta_k$$

$$\alpha_i^+ = \frac{\sum_{t=1}^N P i | x_t - \mu_i^+}{\sum_{t=1}^N x_t P i | x_t} / \frac{\sum_{t=1}^N P i | x_t}{\sum_{t=1}^N P i | x_t}$$

$$\Sigma_i^+ = \frac{\sum_{t=1}^N P i | x_t x_t - \mu_i^+ x_t - \mu_i^+ T}{\sum_{t=1}^N P i | x_t} ,$$

$$P i | x_t = \alpha_i p x_t | \theta_i / \sum_{i=1}^k \alpha_i p x_t | \theta_i \dots b b$$

$$b \dots b$$

$$J \dots \mathbf{Y} \mathbf{Y}$$

$$J \dots \mathbf{Y} \mathbf{Y} \dots x \in X \subset \mathcal{R}^d \dots y \in Y \subset \mathcal{R}^m \dots p x, y \dots p x p y | x \dots q x, y \dots q y q x | y \dots D_x \{x_1, \dots, x_n\} \dots p y | x \dots p x q x | y \dots q y \dots b \dots \mathbf{Y} \mathbf{Y}$$

$$H p \| q = \int p y | x p x \dots q x | y q y \dots dx dy .$$

$$y \{ \dots, k \} \subset R \dots y \dots \mathbf{Y} \mathbf{Y}$$

$$p x = p_0 x = \frac{1}{N} \sum_{t=1}^N G x - x_t \dots p y = i | x = \alpha_i q x | \theta_i / q x | \Theta_k \dots$$

$$q x | \Theta_k = \sum_{i=1}^k \alpha_i q x | \theta_i \dots q y = q y = i \dots \alpha_i > \dots \sum_{i=1}^k \alpha_i \dots ,$$

$G$  .....  $q x|y$  .....  $i$  .....  $q x|\theta_i$  .....  
 $I$  .....  $q y$  .....  $b b$  .....  $q x|y$  .....  
 $p y|x$  .....  $q y$  .....  $q x|y$  .....



$$\alpha_i \cdot b \quad \epsilon > \quad b$$

$\forall y$

Step 1:  $J \quad k \quad \Theta_k$   
 $b \quad l$

Step 2:  $J \quad \Theta_k \quad l$

Step 3:  $J \quad \Theta_k \quad l \quad b$   
 $\Theta'_{k-1} \quad l \quad \Theta'_{k+1} \quad l$

Step 4:  $Acc \ M \quad J \ \Theta'_{k-1} \quad l \quad - J \ \Theta_k \quad l \quad J \quad Acc \ M >$   
 $b \ \Theta'_{k-1} \quad l$

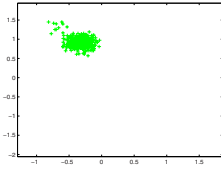
Step 5:  $Acc \ S \quad J \ \Theta'_{k+1} \quad l \quad - J \ \Theta_k \quad l \quad J \quad Acc \ S >$   
 $b \ \Theta'_{k+1} \quad l$

Step 6:  $\epsilon >$   
 $l \quad l \quad k$

$$b \quad b \quad b$$

$J \quad \forall y \quad ff$

$$b \quad b \quad b \quad k \quad b$$



b

b

y y yy

b

b

b

b

b

b

k

b

N

1. r 7 p, .A.: s r b o p r o b s p s r p 7 . ss o p d s r p 7 .  
p: p d, b (d) s r b o p r o b s p s r p 7, pp. 45 72. A d  
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o p r o d s . r o r s 15, 1231 1237 (2002)
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r b A o d o d b o p . r o o p p 7 56, 481 487 (2004)



- 10. ... r OD ... r : A d p r d -  
 p p d o p d o p s . r ro ss p -  
 rs 24(1), 19 40 (2006)
- 11. ... b A p p p A or b for ss p r  
 b A o d o d o p . p o p od 40, 2029 2037 (2007)
- 12. ... A s d p o p r OD p p A or b OD ss p  
 r b A o d o d o p . p o p od rs 29(6), 701  
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- 13. b p , b p , , : o p A or b for p r o d -  
 s . p o p od 37, 131 144 (2004)
- 14. b p , b p , p , , : A or b s for ss p r s b  
 p - p - p p r OD . p o p od 36, 1973 1983 (2003)