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$k: \quad , S: k \quad , \quad S \quad \text{广群 (groupoid)} \quad :$

- $S \quad G,$
- $b, s : G \rightarrow S,$
- $S \times S \quad \circ : G \times_{sSb} G \rightarrow G.$

$k \quad T,$

$(S(T) = \text{Hom}(T, S), G(T) = \text{Hom}(T, S), s, b, \circ)$

$, \quad S(T) \quad , \quad G(T) \quad .$

$: \quad S \times S \quad \varepsilon : S \rightarrow G$

“ $-1$ ” :  $G \rightarrow G,$

$S$   $G$  表示  $V \in \text{Qcoh}(S)$ ,  $G$  .  
 $G$   $u : s^*V \xrightarrow{\sim} b^*V$  :

- $\varepsilon^*u$  ,

- $u \circ : G \times_{s, S^b} G \rightarrow G$   $\text{pr}_1^*u \circ \text{pr}_2^*u$ .

$S$   $T$ ,  $V$  ,  
 $(S(T), G(T), s, b, \circ)$  :  
 $t : T \rightarrow S$   $t^*V$ ,  
 $g : G \rightarrow S$   $g^*u : s(g)^*V \rightarrow b(g)^*V$ .

$S$   $G$  **表示**  $V \in \text{Qcoh}(S)$ ,  $G$   $G \bullet$

## gerbe

$S$  ,  $\text{Sch}/k$  ,  
 (prestack). fpqc  $\mathcal{G}_{S:G}$ .  
 $\text{Sch}/k$   $\mathcal{F}$ ,  
 $\text{Sch}/k$   $\mathcal{F} \rightarrow \text{Qcoh}/k$ .  
 $: k$   $T$ ,  $\mathcal{F}(T) \rightarrow \text{Qcoh}(T)$ .  
 $\text{Rep}(\mathcal{G}_{S:G}) \simeq \text{Rep}(S : G)$ .

## gerbe

$S$  ,  $Sch/k$  ,  $Sch/k$   
 (prestack). fpqc  $\mathcal{G}_{S:G}$ .

$Sch/k$   $\mathcal{F}$ ,  
 $Sch/k$   $\mathcal{F} \rightarrow Qcoh/k$ .  
 $: k$   $T$ ,  $\mathcal{F}(T) \rightarrow Qcoh(T)$ .  
 $, Rep(\mathcal{G}_{S:G}) \simeq Rep(S : G)$ .

gerbe

$\mathcal{G}_{S:G}$  gerbe  
 $(s, b) : G \rightarrow S \times S$  fpqc  
 $G$   
 $G$   $S$



Deligne 1990

$k$  ,  $k$ -张量范畴 (tensor category)  
 $k$ -  $\mathcal{C}$   $\otimes : \mathcal{C} \times \mathcal{C} \rightarrow \mathcal{C}$  :

①  $(\mathcal{C}, \otimes)$   $\mathbb{1}$ .

②  $X \in \mathcal{C}$ ,  $X^\vee$   $\delta : \mathbb{1} \rightarrow X^\vee \otimes X$

$\text{ev} : X \otimes X^\vee \rightarrow \mathbb{1}$  :

$X \xrightarrow{\text{id} \otimes \delta} X \otimes X^\vee \xrightarrow{\text{ev} \otimes \text{id}} X$  和  $X^\vee \xrightarrow{\delta \otimes \text{id}} X^\vee \otimes X \xrightarrow{\text{id} \otimes \text{ev}} X^\vee$

③  $\mathcal{C}$

④  $k \xrightarrow{\sim} \text{Hom}(\mathbb{1}, \mathbb{1})$

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- (1)  $\mathcal{T}, \mathcal{T}', F: \mathcal{T} \rightarrow \mathcal{T}'$   
张量函子 ( $\otimes$ -functor)

- $F(X) \otimes F(Y) \rightarrow F(X \otimes Y),$
- ,

- $\mathcal{T}$  纤维函子 (fiber functor)

$$F: \mathcal{T} \rightarrow \text{Qcoh}(S) \quad S \text{ } k\text{-}$$

- $\mathcal{T}$  淡中范畴 (tannakian category)

$$\omega: \mathcal{T} \rightarrow \text{Qcoh}(S) \quad S \neq \emptyset.$$

$$u: \mathcal{T} \rightarrow S \text{ } k\text{-}, \quad \omega: \mathcal{T} \rightarrow \text{Qcoh}(S)$$

$$u^* \omega: \mathcal{T} \rightarrow \text{Qcoh}(T)$$

 $\mathcal{T}$ 

$$\omega: \mathcal{T} \rightarrow \text{Vect}(K), \quad K \text{ } k$$



. (Deligne 1990)

$\mathcal{T}: k \quad , \omega : \mathcal{T} \rightarrow \text{Qcoh}(S): \quad , \quad S/k \neq \emptyset.$

- $\underline{\text{Aut}}_k^\otimes(\omega) \quad S \times S \quad .$
- $\omega \quad \mathcal{T} \xrightarrow{\sim} \text{Rep}(S : \underline{\text{Aut}}_k^\otimes(\omega)).$
- $\quad , \quad G \quad S \neq \emptyset \quad ,$   
 $S \times S \quad .$   
 $\omega \quad \text{Rep}(S : G) \quad G \quad ,$   
 $\omega \quad : G \xrightarrow{\sim} \underline{\text{Aut}}_k^\otimes(\omega).$

. (Barr-Back)

$$(L, R) \quad , \quad LR \rightarrow \text{id}, \text{id} \rightarrow RL, \\ F = LR, \quad LA \rightarrow LRLA = F(LA) \quad LA \quad F \quad .$$

$$L : \mathcal{A} \rightarrow \mathcal{B} \quad (f, g) : A \rightrightarrows A' \\ (Lf, Lg) \quad , \quad K = \ker(f, g) \quad LK = \ker(Lf, Lg). \\ L \quad \mathcal{A} \quad F \quad \mathcal{B} \quad .$$

$${}_A M_B \quad (A, B) \quad , \quad A \quad , \quad B \\ , \quad B \quad {}_B M_A^\vee \quad (B, A) \quad .$$

$$L : E \rightarrow E \otimes_A {}_A M_B \quad A \quad B \quad , \\ R : F \rightarrow F \otimes_B {}_B M_A^\vee, \quad \text{Barr-Back} \quad , \\ A \quad {}_B M_A^\vee \otimes_A {}_A M_B \quad B \quad .$$

$( \quad )$ 

$$\begin{aligned}
 k: \quad & \quad , B_1, B_2: k \quad . \omega_i : \mathcal{C} \rightarrow (B_i)_{\text{ptf}} \quad , \\
 & (B_i)_{\text{ptf}} \quad B_i \quad . \\
 & : (B_1, B_2) \quad L_k(\omega_1, \omega_2) \quad : \\
 & \quad X \in \mathcal{C}, \quad \omega_1(X)^\vee \otimes \omega_2(X) \rightarrow L_k(\omega_1, \omega_2) \\
 & \quad X, Y \in \mathcal{C}, \quad . \\
 & : \quad . \\
 & \quad L_k(\omega_1, \omega_3) \rightarrow L_k(\omega_1, \omega_2) \otimes L_k(\omega_2, \omega_3). \\
 & . \\
 & L_k(\omega) = L_k(\omega, \omega), \quad . \\
 & , \omega(X)^\vee \otimes \omega(X) \rightarrow L_k(\omega) \quad \omega(X) \rightarrow \omega(X) \otimes L_k(\omega) \\
 & L_k(\omega) \quad \omega(X) \quad .
 \end{aligned}$$

$( )$ 
 $k$  ,  $\mathcal{A}$   $k$ -  $Abel$  ,

 $\text{Hom}$  .

 $k$   $B$ ,  $\omega : \mathcal{A} \rightarrow (B)_{\text{ptf}}$  ,  $\mathcal{B}$   $X$

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$$Y \in \langle X \rangle \quad \langle Y \rangle \subseteq \langle X \rangle.$$

$$B, \quad L_k(\omega|\langle Y \rangle) \rightarrow L_k(\omega|\langle X \rangle),$$

A

A

 $\langle X \rangle$ 

,

 $L_k(\omega)$  $L_k(\omega|\langle X \rangle)$ 

A

,  $\omega$ 

,

$$L_k(\omega_1, \omega_2) \otimes L_k(\omega_1, \omega_2) \rightarrow L_k(\omega_1, \omega_2).$$

A

 $L_k(\omega_1, \omega_2)$ 

B

 $\text{Spec } L_k(\omega_1, \omega_2)$ 

,

 $\underline{\text{Hom}}_S^\otimes(\omega_1, \omega_2),$ 

$$S = \text{Spec } B.$$



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.

$$\Lambda_{\mathcal{T}}(T_1, T_2) \quad \mathcal{T} \quad T_i : \mathcal{C} \rightarrow \mathcal{T},$$

$$T_1(X)^\vee \otimes T_2(X) \quad (\text{coend}), \quad \text{Ind}(\mathcal{T})$$

$$\Lambda_k(T_1, T_2) = \Lambda_{\mathcal{T} \otimes \mathcal{T}}(T_1 \times \mathbb{1}, \mathbb{1} \times T_2).$$

$$T : \mathcal{T} \otimes \mathcal{T} \rightarrow \mathcal{T}, \quad T(\Lambda_k(\text{id}, \text{id})) = \Lambda_{\mathcal{T}}(\text{id}, \text{id}),$$

$$\omega(\Lambda_k(\text{id}, \text{id})) = L_k(\omega).$$

$$\text{ev} : X^\vee \otimes X \rightarrow \mathbb{1} \quad \Lambda_{\mathcal{T}}(\text{id}, \text{id}) \rightarrow \mathbb{1},$$

$$\mathbb{1} \rightarrow \Lambda_{\mathcal{T}}(\text{id}, \text{id})$$

.

$$\mathcal{T} \quad S = \text{Spec } B \quad B \rightarrow \mathcal{F} \leftarrow \leftarrow \mathcal{T} \leftarrow \in$$





# Grassmannian

$G_{\mathcal{O}}$ ,  $G_F$  Ind-  
 $Gr_G = G_F/G_{\mathcal{O}}$ . Ind-

$G$   $GL_n$ ,  $Gr_G$   $Gr_{GL_n}$   
 $GL_n$ ,  $Gr_{GL_n}$   
 Grassmannian

$Gr_G$  : Beauville-Laszlo  
 $X$ ,  $x \in X$ ,  $X^* = X \setminus \{x\}$ .

$R$ ,  $X_R = X \otimes_k R$ .  
 $Gr_G(R) = (\mathcal{E}, \beta)$   $\mathcal{E}|_{X_R}$   $G$ - (torsor), isom  
 $\beta : \mathcal{E}|_{X_R^*} \rightarrow \mathcal{E}^{\circ}|_{X_R^*}$

$G_{\mathcal{O}}$ 

$ev : G_{\mathcal{O}} \rightarrow G$       $t = 0$      .  
 $(B, T)$      Borel     ,      $I = ev^{-1}B$       $G_F$      Iwahori     ,  
 $G_{\mathcal{O}}$      Parahori     .     Cartan     :  
 $G_{\mathcal{O}} \backslash G_F / G_{\mathcal{O}} = W \backslash (X_{\bullet} \times W) / W = X_{\bullet}^+$  .

$\lambda \in X_{\bullet}$  ,      $\mathbb{G}_m \rightarrow T$       $\mathbb{G}_{m,F} \rightarrow T_F$  .  
 $t^{\lambda}$       $t \in \mathbb{G}_{m,F}$      .

$G_{\mathcal{O}}$       $Gr_G$      ,      $X_{\bullet}^+$      .  
 $\lambda \in X_{\bullet}^+$  ,  $G_{\mathcal{O}} t^{\lambda} G_{\mathcal{O}} / G_{\mathcal{O}}$       $G_{\mathcal{O}}$      .

$Gr_G^{\lambda} = G_{\mathcal{O}} t^{\lambda} G_{\mathcal{O}} / G_{\mathcal{O}}$      ,  
 $Perv_{G_{\mathcal{O}}}(Gr_G, A)$       $IC_{Gr_G^{\lambda}}$      .

$$Gr_G \times Gr_G \xleftarrow{p} G_F \times Gr_G \xrightarrow{q} G_F \times^{G_O} Gr_G = Gr_G \otimes Gr_G \xrightarrow{m} Gr_G.$$

$$\mathcal{F}, \mathcal{G} \in \text{Perv}_{G_O}(Gr_G, A),$$

$$Gr_G \otimes Gr_G$$

$$\mathcal{F} \boxtimes \mathcal{G} \quad p^*(\mathcal{F} \boxtimes \mathcal{G}) = q^*(\mathcal{F} \boxtimes \mathcal{G}).$$

$$\mathcal{F} \star \mathcal{G} = m_*(\mathcal{F} \boxtimes \mathcal{G}), \quad \mathcal{F} \quad \mathcal{G} \quad .$$

$$Gr_G \times Gr_G \xleftarrow{p} G_F \times Gr_G \xrightarrow{q} G_F \times^{G_O} Gr_G = Gr_G \otimes Gr_G \xrightarrow{m} Gr_G.$$

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$$\mathcal{F} \star \mathcal{G} = m_*(\mathcal{F} \boxtimes \mathcal{G}), \quad \mathcal{F} \quad \mathcal{G} \quad .$$

$$\text{Perv}_{G_{\mathcal{O}}}(Gr_G, A)$$

$$\lambda \in X_{\bullet}^+, \quad IC_{\lambda} = IC_{\overline{Gr_G^{\lambda}}}.$$

$$\text{Hom}(IC_{\lambda}, IC_{\mu}[1]) = 0$$

$$\lambda, \mu \in X_{\bullet}^+.$$

$$:$$

- $\overline{Gr_G^{\lambda}} = \bigsqcup_{\nu \leq \lambda} Gr_G^{\nu}$ ,  $\dim \overline{Gr_G^{\lambda}} = \dim Gr_G^{\lambda} = \langle 2\rho, \lambda \rangle$ ,
- $\mathcal{H}^i(IC_{\lambda}) = 0$   $i < \langle 2\rho, \lambda \rangle$ .



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• :  $G_F$  Kac-Moody ,  $G_O$   
Parahori , .

• :  $Gr_G^\lambda$  ,  $t^\lambda$  .

• :  $C_{\lambda, \alpha} \setminus \{t^{\lambda - \alpha^\vee}\} \subset Gr_G^\lambda$  ,  $C_{\lambda, \alpha^\vee}$  ,  
 $\alpha$  ,  $\lambda, \lambda - \alpha^\vee \in X_\bullet^+$  ,  
 :  $SL_2$  ,



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Bruhat ,  $Gr_G = \sqcup_{w \in \widetilde{W}/W} IwG_{\mathcal{O}}/G_{\mathcal{O}}$ .

$w_\lambda \in Wt^\lambda W$  ,  $Iw_\lambda G_{\mathcal{O}}/G_{\mathcal{O}}$  ,  $Gr_G^\lambda$  .

$IC_\lambda = IC_{Iw_\lambda G_{\mathcal{O}}/G_{\mathcal{O}}}$ .

$Fl_G = G_F/I \rightarrow Gr_G$ ,  $G/B^-$  .

$Iw_\lambda G_{\mathcal{O}}/G_{\mathcal{O}}$  ,  $Iw_\lambda w_o I/I$ .

,  $IC_{IwI/I}$  ,  $\overline{IwI/I}$

Bott-Samelson ,



$$\mathcal{F}, \mathcal{G} \in \text{Perv}_{G_{\mathcal{O}}}([()]/211111/11220)$$



$$U = [B, B] \quad B, \quad \lambda \in X_{\bullet},$$

$$S_{\lambda} = U_F t^{\lambda} G_{\mathcal{O}} / G_{\mathcal{O}} \quad U_F \quad Gr_G$$

:

- $\overline{S_{\lambda}} = \sqcup_{\nu \leq \lambda} S_{\nu};$

- $2\rho^{\vee} \quad \mathbb{G}_m \rightarrow T \hookrightarrow G$   
 $\mathbb{G}_m \quad Gr_G$

$$S_{\lambda} = \{ x \in Gr_G \mid \lim_{s \rightarrow 0} 2\rho^{\vee}(s) \cdot x = t^{\lambda} \}.$$

$$U_F^{-}$$

$$T_{\lambda} = \{ x \in Gr_G \mid \lim_{s \rightarrow \infty} 2\rho^{\vee}(s) \cdot x = t^{\lambda} \}.$$

# ( )-Braden's hyperbolic localization theorem

$$\begin{array}{l}
 \mathbb{G}_m \text{ action on } X, \quad \rho: \mathbb{G}_m \times X \rightarrow X, \\
 S \in \mathcal{D}(X), \\
 \rho^* S = L \boxtimes S, \quad L \in \mathcal{D}(\mathbb{G}_m).
 \end{array}$$

(Braden 2003)

$$\begin{array}{l}
 X \text{ is } \mathbb{G}_m \text{-contractible. } F = X^{\mathbb{G}_m} \\
 X^+ = \{x \in X \mid \lim_{s \rightarrow 0} s \cdot x \text{ exists}\}, \quad X^- = \{x \in X \mid \lim_{s \rightarrow \infty} s \cdot x \text{ exists}\}.
 \end{array}$$

$$f^\pm: F \rightarrow X^\pm, g^\pm: X^\pm \rightarrow X, \pi^\pm: X^\pm \rightarrow F.$$

$$S \in \mathcal{D}(X) \quad \mathbb{G}_m \text{-equivariant, } :$$

- $(f^\pm)^* S = (\pi^\pm)_* S, (f^\pm)! S = (\pi^\pm)! S,$
- $(f^-)^*(g^-)! S \rightarrow (f^+)^!(g^+)^* S.$

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$$\overline{Gr_G^\lambda} \quad ,$$

$$H_{T_\mu}^k(Gr_G, IC_\lambda) \xrightarrow{\sim} H_c^k(S_\mu, IC_\lambda).$$

$$k \neq \langle 2\rho, \mu \rangle \quad .$$

$$\lambda \in X_{\bullet}^+, \mu \in X_{\bullet}$$

$$\dim \overline{Gr_G^\lambda} \cap S_\mu = \langle \rho, \lambda + \mu \rangle.$$

$$\mathcal{F} \in \text{Perv}_{G_O}(Gr_G)$$

$$H_c^k(S_\mu, \mathcal{F}) = 0, k > \langle 2\rho, \mu \rangle.$$

$$H_{T_\mu}^k(Gr_G, \mathcal{F}) = 0, k < \langle 2\rho, \mu \rangle.$$



$( )$ 
 $: \mathcal{F} \in \text{Perv}_{G_O}(Gr_G)$ 

- $H_c^{\langle 2\rho, \mu \rangle}(S_\mu, \mathcal{F}) = H_c^{\langle 2\rho, \mu \rangle}(\overline{S}_\mu, \mathcal{F}),$
- $H_c^{\langle 2\rho, \nu \rangle}(\overline{S}_\mu, \mathcal{F}) = H_c^{\langle 2\rho, \nu \rangle}(\overline{S}_\nu, \mathcal{F}), \nu \leq \mu,$
- $H_c^{\langle 2\rho, \mu \rangle + \text{odd}}(\overline{S}_\mu, \mathcal{F}) = 0.$

$$T_\mu, \quad F_\mu = H_c^{\langle 2\rho, \mu \rangle}(S_\mu, -), \quad H(Gr_G, \mathcal{F}) = \bigoplus_{\mu \in X_\bullet} F_\mu(\mathcal{F}).$$

$$H_{T_\mu}^{\langle 2\rho, \mu \rangle}(Gr_G, \mathcal{F}) \quad H^{\langle 2\rho, \mu \rangle}(Gr_G, \mathcal{F}) \quad H^{\langle 2\rho, \mu \rangle}$$

$$Gr_{G,X}(R) = \left( \begin{array}{l} \text{Grassmannian} \\ X, \quad Gr_{G,X} \\ (x, \mathcal{E}, \beta) \end{array} \right), \quad \text{isom}$$

$x \in X(R), \mathcal{E} \text{ } X_R \text{ } G$

$\beta : \mathcal{E}|_{X_R \setminus \Gamma_x} \rightarrow \mathcal{E}^\circ|_{X_R \setminus \Gamma_x}$

$Gr_{G,X^2}$ , Beilinson-Drinfeld Grassmannian.

- $Gr_{G,X^2}|_{X^2 \setminus \Delta} = (Gr_{G,X} \times Gr_{G,X})|_{X^2 \setminus \Delta}$ .
- $Gr_{G,X^2}|_{\Delta} \simeq Gr_{G,X}$ .



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$$U = X^2 \setminus \Delta, \quad Gr_{G,X} \otimes Gr_{G,X} \xrightarrow{m} Gr_{G,X^2}.$$

$$X = \mathbb{A}^1, \quad Gr_{G,X} = Gr_G \times X, \\ \mathcal{F} \in \text{Perv}_{G_O}(Gr_G), \quad \mathcal{F}_X \in \text{Perv}_{G_O,X}(Gr_{G,X}).$$

$$U, \quad \Delta, \quad \mathcal{F}_X \star_X \mathcal{G}_X. \\ \mathcal{F} \star \mathcal{G}[2], \\ U, \quad \Delta, \quad \mathcal{F} \boxtimes \mathcal{G}[2].$$

$$Gr_{G,X^2}$$

$$\mathbb{G}_m, \quad Gr_{G,X} \otimes Gr_{G,X}, \quad (t^{\mu_1}, t^{\mu_2})_{X^2}. \\ m, \quad \Delta, \\ \mu \in X \bullet, \quad S_\mu(X^2).$$

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$$\begin{array}{ccc}
 S_\mu(X^2) & \Delta & \\
 U & & \\
 & & \text{F } S_\mu, \\
 & & S_{\mu_1} \times S_{\mu_2}. \\
 & & \mu_1 + \mu_2 = \mu \\
 & & \mathcal{H}_c^{\langle 2\rho, \mu \rangle - 2}(S_\mu(X^2)/X^2, \mathcal{F}_X \star_X \mathcal{G}_X). \\
 \mathcal{F}_X \star_X \mathcal{G}_X & & \\
 \Delta & & \\
 U & & \\
 H^{\langle 2\rho, \mu \rangle} & \text{F} & \\
 & S_{\mu_1} \times S_{\mu_2}, \mathcal{F} \boxtimes \mathcal{G} & = \text{L} \\
 & \mu_1 + \mu_2 = \mu & F_{\mu_1}(\mathcal{F}) \otimes F_{\mu_2}(\mathcal{G}).
 \end{array}$$

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$$\begin{array}{ccc}
 S_\mu(X^2) & \Delta & \\
 U & & \\
 & & \text{F } S_\mu, \\
 & & S_{\mu_1} \times S_{\mu_2}. \\
 & & \mu_1 + \mu_2 = \mu \\
 & & \\
 \mathcal{F}_X \star_X \mathcal{G}_X & & \mathcal{H}_c^{\langle 2\rho, \mu \rangle - 2}(S_\mu(X^2)/X^2, \mathcal{F}_X \star_X \mathcal{G}_X). \\
 \Delta & & \\
 U & & H_c^{\langle 2\rho, \mu \rangle}(S_\mu, \mathcal{F} \star \mathcal{G}) = F_\mu(\mathcal{F} \star \mathcal{G}). \\
 & & \\
 H^{\langle 2\rho, \mu \rangle} & \text{F} & \\
 & & S_{\mu_1} \times S_{\mu_2}, \mathcal{F} \boxtimes \mathcal{G} = \text{L} \\
 & \mu_1 + \mu_2 = \mu & \mu_1 + \mu_2 = \mu \\
 & & F_{\mu_1}(\mathcal{F}) \otimes F_{\mu_2}(\mathcal{G}).
 \end{array}$$

- $X^2$
- $\mathcal{H}^k(Gr_{G, X^2}, \mathcal{F}_X \star_X \mathcal{G}_X)$

$$\text{Sat}_G = \text{Perv}_{G_0}(Gr_G).$$

$$F = H(Gr_G, -) \quad \text{Sat}_G$$

$$\text{SVect}(A),$$

$$\text{Vect}(A)$$

$$\text{Sat}_G \quad \text{Rep}(\mathcal{G}, A).$$

$$\text{Sat}_G$$

$$, \quad \mathcal{G}$$

$$\text{Sat}_G$$

$$X, \quad \langle X \rangle$$

$$G$$

$$\text{Sat}_G$$

$$\mathcal{G}$$

$$\mathcal{G} \quad \mathcal{G}$$

$( \quad )$ 

$$\begin{array}{l}
 T \quad \mathbb{G}_m^n, \\
 \text{Gr}_T \quad X_\bullet(T) \\
 \text{Sat}_T \quad X_\bullet(T) \\
 \mathcal{P} = \mathcal{P} \\
 F = \prod_{\mu \in X_\bullet} F_\mu \\
 \text{Sat}_G \rightarrow \text{Vect}_{X_\bullet}(A) \rightarrow \text{Vect}(A). \\
 \mathcal{G} \quad X_\bullet^+, \quad \mathcal{G}
 \end{array}
 \quad : \quad
 \mathcal{P} \rightarrow \mathcal{G}.$$

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$2\rho \in X^\bullet = \hat{X}_\bullet,$   
 $\mathcal{G}$  , Borel ,  $\mathfrak{p}.$   
 $\lambda \in X_\bullet^+, IC_\lambda$   $\lambda$   $F(IC_\lambda)$   $X_\bullet^+.$   
 $X_\bullet^+ = \mathcal{G}^\bullet.$   
 $F(IC_\lambda)_\mu$   $\mu \leq \lambda.$   $\hat{X}_\bullet$   
 (root lattice)  $\mathcal{Q}_+$   $Q_+^\vee.$   
 $\mathcal{G}$  .

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• Zeta

## Hodge

$$\begin{array}{l}
 k: \quad 0 \quad , \quad \bar{k}, \text{ Galois} \quad \Gamma = \text{Gal}(\bar{k}/k). \\
 X: k \quad . \quad \ell, H_\ell(X): X \xrightarrow{\ell} \mathbb{Q} \quad , \\
 H_{\text{dR}}(X): X \quad \text{de Rham} \quad . \quad H_{\mathbb{A}}(X) = H_{\text{dR}}(X) \times_{\ell} H_\ell(X). \\
 \sigma : k \hookrightarrow \mathbb{C}, H_\sigma(X) \quad \sigma X = X \otimes_{k, \sigma} \mathbb{C} \quad \text{Betti} \quad . \\
 \quad : H_\sigma(X) \otimes (\mathbb{C} \times \mathbb{A}_f) \xrightarrow{\sim} H_{\mathbb{A}}(\sigma X). \\
 \sigma^* \quad H_\sigma(X) \hookrightarrow H_{\mathbb{A}}(\sigma X).
 \end{array}$$



## Hodge

$$\begin{aligned}
 k: & \quad 0 \quad , \quad \bar{k}, \text{ Galois} \quad \Gamma = \text{Gal}(\bar{k}/k). \\
 X: & \quad k \quad . \quad \ell, H_\ell(X): X \xrightarrow{\ell} \mathbb{Q}^\ell, \\
 H_{\text{dR}}(X): & \quad X \quad \text{de Rham} \quad . \quad H_{\mathbb{A}}(X) = H_{\text{dR}}(X) \times_{\ell} H_\ell(X). \\
 \sigma: & \quad k \hookrightarrow \mathbb{C}, H_\sigma(X) \quad \sigma X = X \otimes_{k, \sigma} \mathbb{C} \quad \text{Betti} \quad . \\
 & \quad : H_\sigma(X) \otimes (\mathbb{C} \times \mathbb{A}_f) \xrightarrow{\sim} H_{\mathbb{A}}(\sigma X). \\
 \sigma^* & \quad H_\sigma(X) \hookrightarrow H_{\mathbb{A}}(\sigma X).
 \end{aligned}$$

$H_{\mathbb{A}}^{2p}(X)(p)$   $t$  **相对于  $\sigma$  的 Hodge 类**

$$\sigma^* t \in H^{2p}(\sigma X, \mathbb{Q})(p) \cap H^{p,p}(\sigma X).$$

$H_{\mathbb{A}}^{2p}(X)(p)$   $t$  **绝对 Hodge 类**

$$\sigma: k \hookrightarrow \mathbb{C} \quad \sigma \quad \text{Hodge} \quad .$$

$$C_{\text{AH}}^p(\quad)$$

$$\mathbf{CV}_k : \mathcal{M}_k \rightarrow \mathcal{M}_k, \quad (h(X), X \in \mathbf{V}_k, \dots)$$

$$\mathrm{Hom}(h(X), h(Y)) = \mathrm{Mor}^0(X, Y).$$

$$h(X \sqcup Y) = h(X) \oplus h(Y), \quad h(X \times Y) = h(X) \otimes h(Y).$$

$$\mathbb{1} = h(\mathrm{pt}).$$

$$\mathbf{CV}_k \text{ (effective) motive } \dot{\mathbf{M}}_k^+.$$

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$$\mathbb{P}^1, \quad h(\mathbb{P}^1) = h^0(\mathbb{P}^1) + h^2(\mathbb{P}^1) = h(\text{pt}) + L.$$

$$H(L) = \mathbb{Q}(-1),$$

$$\text{Hom}(M, N) \xrightarrow{\sim} \text{Hom}(M \otimes L, N \otimes L)$$

motive  $M, N$ .

 $L$ 

:

motive  $\dot{M}_k$  :

①  $(M, m), M \in \dot{M}_k^+, m \in \mathbb{Z};$

②

$$\text{Hom}_{\dot{M}_k}((M, m), (N, n)) = \text{Hom}_{\dot{M}_k^+}(M \otimes L^{r-m}, M \otimes L^{r-n})$$

$r \geq m, n;$

③

$\dot{M}_k^+$

$\dot{M}_k$ 
 $H^r(X)$  Hodge

 $\text{Mor}^0(X, X)$ 
 $\text{Hom}_{\dot{M}_k^+}((h(x), p), (h(x), p)) = p\text{Mor}^0(X, X)p$ 
 $p$ 

Jannsen 1992

Lemma 2



$( )$ 
 $\dot{M}_k$ 
 $(\text{rigid})$ 
 $L$ 
 $h(X), X \in \mathbb{V}_k, X \quad n,$ 

$$h(X)^\vee = h(X)(n),$$

 $\delta \quad [\Delta_X], \quad \text{ev}$ 
 $p \in \text{Hom}(h(X), h(X)),$ 
 ${}^t p \quad h(X)^\vee$ 

$$(h(X), p)^\vee = (h(X)^\vee, {}^t p).$$



$$\dim h(X) = \chi(X), \quad X \in \mathcal{V}_k,$$

$$\dot{\phi} : M \otimes N \rightarrow N \otimes M, \quad \dot{\phi} = \bigoplus \dot{\phi}^{r,s}, \quad \dot{\phi}^{r,s} : M^r \otimes N^s \rightarrow N^s \otimes M^r$$

 $\dot{M}_k$ 

$$\phi : M \otimes N \rightarrow N \otimes M, \quad \phi = \bigoplus \phi^{r,s}, \quad \phi^{r,s} = (-1)^{rs} \dot{\phi}^{r,s}.$$

 $M_k,$ 
**正确的 motive 范畴.**

$$H_\ell, H_{\text{dR}}, H_\sigma$$













xÊ X

—V Š 8 T +» -¿ Óf n- :V! V %<sup>2</sup>  
cÁ XđY Ô « Ú¾”

$$V = \sum_{i=1}^n a_{n-1}^{n-1} + \dots + (-1)^n a_n$$

éÄ a<sub>i</sub> = i` i ^i V X

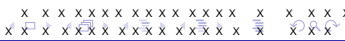
šç X

› ,gç” # /2i V = i` n ^n V X  
1¿• ñ -Ä

$$f_i = \frac{X}{(i B^n i)}$$

$$= S_n / (S_i S_{n-i})$$

a<sub>i</sub> = i` f<sub>i</sub> ^n V :



•^ X

Q} V "î™ O8 <Ä

$$i`f_i \wedge^n V = i` \quad i \wedge^i V \quad /B\mathbb{K} \quad i \quad :$$

$$n \quad i \quad :$$

šç X

ï .2HB,RM2Ny G2KK2 U×K^" © Y

i(B)/= /B\mathbb{K}; i(A);:::; i(A^i) J« Ú¾" X

òç V = 1^m -m 3... fž /B\mathbb{K} = m - UY + N à š

^" , ^ -Rò Ú¾"©, ^ X



P - X

+9%² Š 8 T +» V C; cÁ f 2 >Q(K;V) -

ï •^ i=1 ÕÑwï 2tT1yJ e±ñ

$$2tTi`f V = /2i 2t\mathbb{T} V :$$

xÊ X

—M ©%² ™ O8 T +» -f :M ! M %² cÁ -  
éÄY "ÿèy ,  
X ( 1)<sup>n</sup> i`f n ^n M t<sup>n</sup> X i`f m avKM t<sup>m</sup> = 1:  
n 0 m 0

šç X

1¥3† 'f J• -éQ-šç+`P3M..f I-®Ä  
^nM avk<sup>n</sup>M = ^nM avk<sup>n</sup>M:  
0 n l 0 n l  
n 2p2M n Q//  
öY Y Ö¥ U£àš, > ¥0Ö` © V,  
0!^ lM !^ l 1M avkM ! ! avkM ! 0;  
•w<sup>a</sup>Ëß¥~çF•g "ã -Yšç ©%² 3`ç X



š ø ù v x

+93 ... f n - " p<sub>n</sub> =  $\frac{1}{n!} 2^{S_n}$  ; q<sub>n</sub> =  $\frac{1}{n!} 2^{S_n} (1)^{b;M}$  -  
 ã ð + • a v K y ^ n 3 † ^ « X  
 ð Y £

¿ X X Á 0 Ö ` ã ð q<sub>n</sub> p<sub>m</sub> q<sub>n+1</sub> p<sub>m</sub> y q<sub>n</sub> p<sub>m+1</sub> q<sub>n</sub> p<sub>m</sub> X  
 £ ± ñ [ Ö ` ® 0 X  
 4 ù - ï Ü + f ÿ = œ = [ " , [%<sup>2</sup> @ f e C © 3 † ^ X  
 Â e ... Y - ò , □

- ò X

$$q_n p_m = \frac{n(m+1)}{m+n} q_n p_{m+1} q_n p_m + \frac{m(n+1)}{m+n} q_n p_m q_{n+1} p_m :$$





Ô ` Y x Ê ± ñ

P - X

—V © î Š 8 + » - © V c Á - é È `

x Ê = Ä " è y ,

/ 2 i 1 t V 1 = X i ` n a v K V t n :

n 0

R ò U Y w 2 i á f Ê

$$Z(X;t) = \int_0^n X \cdot i \cdot 6_q^n \cdot a \cdot v \cdot K \cdot h(X) \cdot t^n;$$

$$= \int_0^n X^0 \cdot i \cdot 6_q \cdot h(a \cdot v \cdot K \cdot X) \cdot t^n;$$

n 0



U Y # K Q i B p B á y w 2 i á f 2 0 f " % ^ 2 8 3 t ' f ,

$$Z(X;t) = \sum_{n=0}^{\infty} h(a v K X) t^n$$

$$h p ñ i` h(X) = \sum_i^P (1)^i i` h^i(X) -$$

n ç y n ĩ È ç © , c X

4 - + ` p ê c Á K\_0(J\_k)! R - ® U Y < ' ¿ w 2 i á f X

ò " 2 i i B b = i` i j h ( ) -

Ô « 0 ¿ > Q / ; ? h^p;q ^ X



† ò : ° i i b + ? ' 2 ,

+ 9 % 2 2 n Ó W f T M X - P<sub>x;y</sub>(X) n P

+ f > Q / p 2 ÿ è á f - /

$$P_{x;y}(X) = \sum_{n \ i;j \ n} h^{n+i;n+j}(X) x^i y^j :$$

— S % 2 ¼ μ - S<sup>(n)</sup> = a v K S = S<sup>n</sup> / S<sub>n</sub> -  
 S<sup>[n]</sup> n P S n μ > B H # ' 2 ` i X

$$h^{i;j} = h^{i;j}(S) - \acute{e} \ddot{A} \hat{^} \text{''} ,$$

