第二届全国 PDE 博士生论坛

论坛时间：2019 年 10 月 10 日 -13 日

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1
论坛简介

2019　10　10　　10　13

"　　　PDE　　　"

Einstein

Green　　　Dirichlet

KdV

Euler　　　Navier-Stokes
Monge–Ampère　　　Cauchy-
Riemann　　KdV　　　　Maxwell
Maxwell

NavierStokes　　　Euler
Maxwell　　　　Schrödinger
Yang-Mills　　　　Einstein

19
Harnack
Hilbert　　　　Dirichlet

Sobolev

Hille-Yoshida
De Giorgi-Nash

Matlab　Mathematica
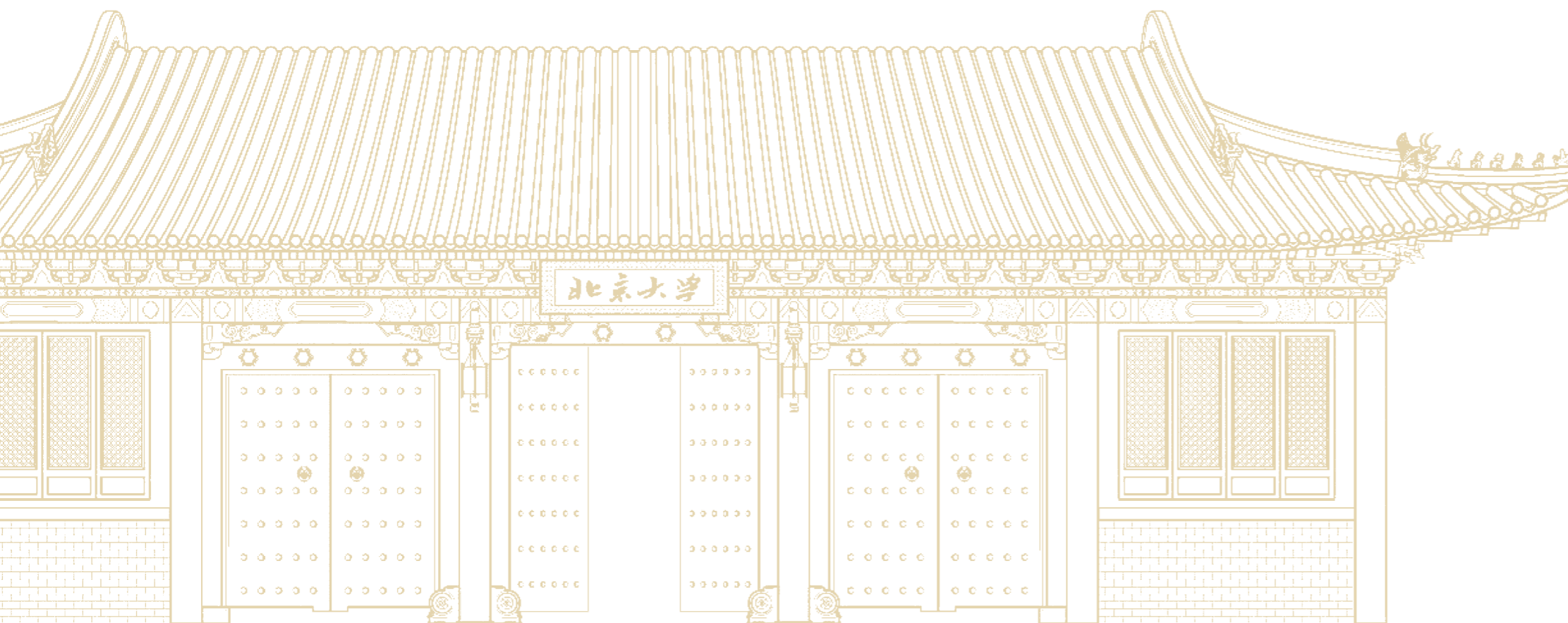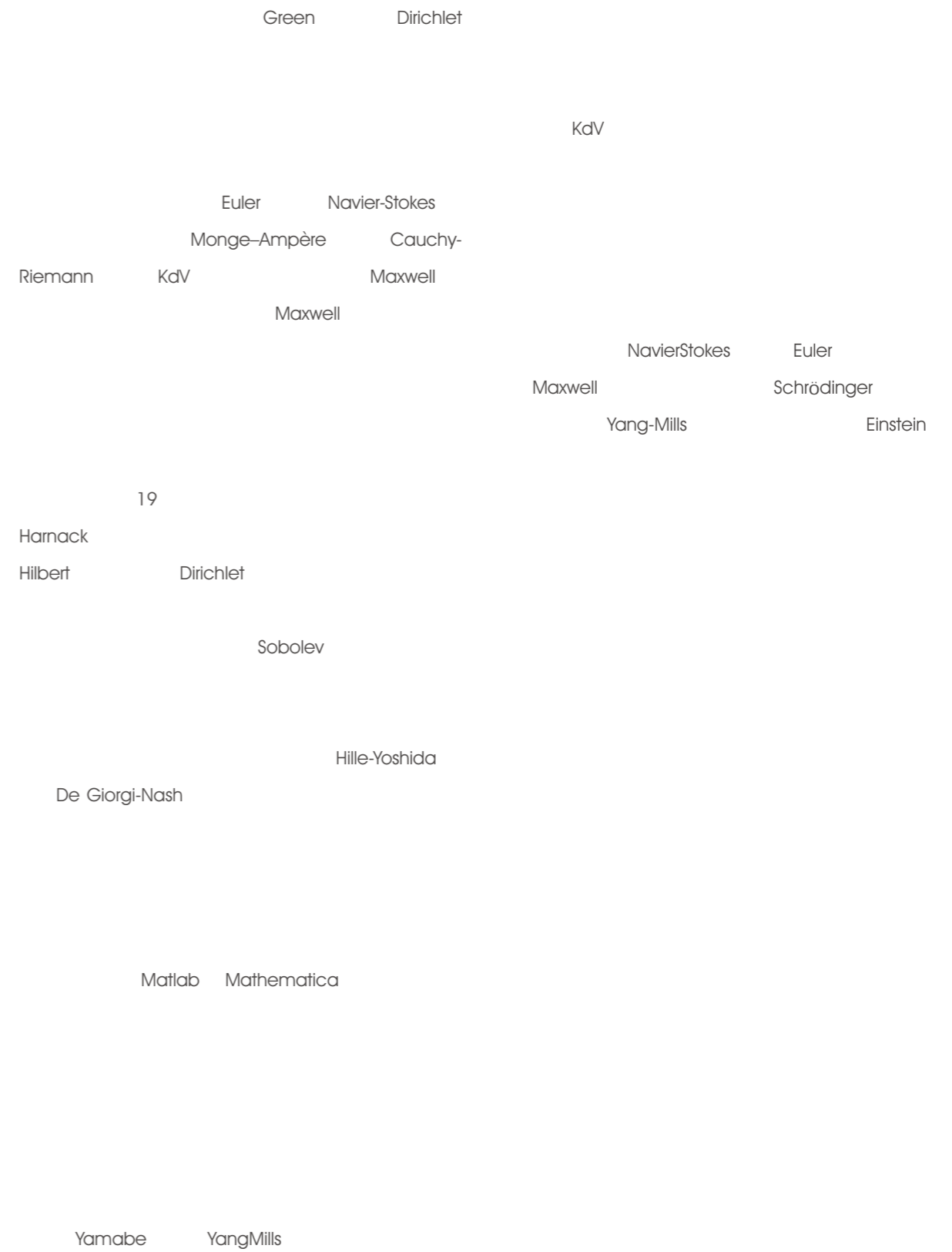
Euler
D′Alembert
Laplace

Yamabe　　YangMills

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实用信息

10   10   13:30-17:00   9
10   11   8:30-9:00

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                        126
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                1 km   4   "   "   1km
        428   /
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1000
            5
        300

1.   32.5km   110

2.   ——   10   ——   4   "   "   10
30

1   21km   70
2   2   ——   4   "   "   10   5

1   28.5km   90
2   4   "   "   10   5

1   18.5km   60
2   9   ——   4   "   "   10   4

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论坛日程（10 月 11 日）

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09:45-10:00			
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			（　　　　）
			: ₂Yamabe problem on conic 4-sphere
11:30-13:30			
14:00-15:45	1560		: Eigenvalues estimates for degenerate elliptic operators on compact manifold
			: Barotropic instability of shear flows and linear inviscid damping for the -plane equation
			: Classical solutions to compressible dissipative elastodynamics with zero shear viscosity
15:45-16:15	1384		
16:15-17:25	1560		: Global attractor of multi-valued operators and its application
			: Liouville type equation with exponential Neumann boundary condition and with singular data
18:00-19:30			

论坛日程（10 月 12 日）

时间	地点	主持人	内容
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			: Improved variable coefficient square functions and local smoothing of Fourier integral operators
			: The Lp Minkowski problem for the electrostatic capacity
10:15-10:45	1384		
10:45-11:55	1114		: Sharp One Component Regularity for Naiver-Stokes
			: Long time asymptotics for chemotaxis with free boundary
12:00-13:30			
14:00-15:45	1114		: Domain Wall Solitons Arising in Classical Gauge Field Theories
			: Well-Posedness and Global Attractor for the Two-dimensional Zakharov-Kuznetsov Equation
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15:45-16:15	1384		
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论坛日程（10 月 13 日）

时间	地点	主持人	内容
9:00-10:10	1114		Exterior rigidity theorems for fully nonlinear elliptic equations
			On the equivalence of viscosity and distribution solutions of second-order PDEs with Neumann boundary conditions
10:10-10:40	1384		
10:40-11:50	1114		The Free Boundary Problem in Incompressible Elastodynamics
			Partial regularity of suitable weak solutions of the Navier-Stokes-Planck-Nernst-Poisson equation
12:00-13:30			

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开幕式议程

2019　10　11　　09　00

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报告信息

2019　10　11　　10　00- 11　30

Global Vanishing Viscosity Limit for Incompressible Viscoelasticity in Two Dimensions

Vanishing viscosity limit is one of the central topics in both the theory of fluid mechanics and the analysis of partial differential equations. In general, it is expected to be true for Cauchy problem locally in time. However, as long as the time is global, the verification of such a theory is highly nontrivial and is thus open for most fluid systems. In this talk, we report our results on two dimensional incompressible viscoelasticity.

$_2$ Yamabe problem on conic 4-sphere

: We give a necessary condition for the existence of conic metrics with positive $_2$ constant curvature on 4-sphere. This is a nonlinear generalization of conic surface theory of Troyanov. Based on the Mass we constrcuted, when singular divisors converges to critical case, we prove that the conic 4-sphere converges to American football in Gromov-Hausdorff topology. This is joint work with Hao Fang.

2019　10　11　　14　00- 15　45

1560

Eigenvalues estimates for degenerate elliptic operators on compact manifold

: Let $X = (X_1, X_2, \cdots, X_m)$ be a system of real smooth vector elds de ned on a smooth compact manifold M which endowed with a smooth positive measure μ . Assume X satis es the Hörmander′s condition, then the formally self-adjoint operator $\Delta_x = -\Sigma_{(i=1)}^m X_i^* X_i$ is a sub-elliptic operator. Denote $_k$ be the k-th eigenvalue for the sub-elliptic operator $_x$ on M. We shall give a lower bound of $_k$ and also establish an explicit asymptotic formula of $_k$ under a certain condition. Actually, this condition is necessary for such asymptotic result to be hold. Thus it can be obviously deduced that our lower bound estimates for $_k$ is optimal in the sense of the order of k.

Barotropic instability of shear flows and linear inviscid damping for the -plane equation

: In this talk, we consider barotropic instability and linear damping around shear flows for incompressible fluids with Coriolis effects. For a class of shear flows, we discuss the method, based on Hamiltonian structure and index theory, to find the sharp stability condition. We will apply this method to the Sinus flow. The addition of the Coriolis force is found to bring some fundamental changes. Then we discuss the difference from no Coriolis effects, in the proof of linear damping by establishing the limiting absorption principle, which is based on the compactness method. We give the explicit decay rate of the velocity for a class of monotone shear flows by the method based on the space-time estimate and the vector field method. We also prove the existence of nontrivial traveling wave solutions near shear flows with non-resonant neutral modes, which are purely due to Coriolis effects.

: Classical solutions to compressible dissipative elastodynamics with zero shear viscosity

: This talk will focus on the compressible dissipative elastodynamic system with zero shear viscosity in 2D. The global stability around a constant equilibrium is given. Due to the low pointwise dispersive estimate for the divergence-free projection of solutions, and the lack of null structures in convective terms, energy estimates, combined with Green's functions and dispersive estimates, for both solutions and their potentials are obtained to control the interactions in nonlinear terms. Some related results will be discussed. This is a joint work with Prof. Xianpeng Hu.

2019 10 11   16 15- 17 25

1560

Global attractor of multi-valued operators and its application

In this talk, we are concerned with the existence of global attractors for a few classes of multi-valued operators. We establish some criteria and give their applications to a strongly damped wave equation with fully supercritical nonlinearities and without the uniqueness of solutions. Moreover, the geometrical structure of the global attractors of the corresponding multi-valued operators is shown.

Liouville type equation with exponential Neumann boundary condition and with singular data

: In this paper we will analyze the blow-up behaviors of solutions to the singular Liouville type equation with exponential Neumann boundary condition. We generalize the Brezis–Merle type concentration-compactness theorem to this Neumann problem. Then along the line of the Li–Shafrir type quan-

tization property we show that the blow-up value m(0) 2 N {2 (1+)+2 (N {0})} if the singular point 0 is a blow-up point. In the end, when the boundary value of solutions has an additional condition, we can obtain the precise blow-up value m(0)=2 (1+).

2019 10 12   8:30- 10:15

1114

Global solutions of 3-D Navier-Stokes system with small unidirectional derivative

: Given initial data $u_0 = \left(u_0^h, u_0^3\right) \in H^{\frac{1}{2}}(R^3) \cap B_{2,1}^{0,\frac{1}{2}}(R^3)$ with u_0^h belonging to $L^2(R^3) \cap L^\infty\left(R_v; H^{-\delta}(R_h^2)\right) \cap L^\infty\left(R_v; H^3(R_h^2)\right)$ for some]0,1[, if in addition $\partial_3 u_0$ belonging to the homogeneous anisotropic Sobolev space, $H^{\frac{1}{2},0}$ we prove that the classical 3-D Navier-Stokes system has a unique global Fujita-Kato solution pro- vided that the $H^{-\frac{1}{2},0}$ norm of $\partial_3 u_0$ is sufficiently small compared to exp (-C(A (u_0^h)+B (u_0))) with A (u_0^h) and B (u_0) being scaling invariant quantities of the initial data, and which is scaling invariant with respect to the variable x_3. This result provides some classes of large initial data which are large in Besov space $B_{\infty,\infty}^{-1}$ and which generate unique global solutions to 3-D Navier-Stokes system. In particular, we extend the previous results in a series of works by Chemin, I. Gallagher et al for initial data with a slow variable to multi-scales slow variable initial data.

Improved variable coefficient square functions and local smoothing of Fourier integral operators.

Local smoothing conjecture which was formulated by Sogge has close relationship with other significant conjectures in Harmonic analysis and finds its extensive applications in PDEs. In this talk, we will present the recent improvement of local smoothing estimate of a certain class of Fourier integral operators satisfying cinematic curvature conditions. The main ingredients in our proof are bilinear oscillatory integral estimate, multilinear oscillatory integral estimate and variable coefficient decoupling inequality.

The Lp Minkowski problem for the electrostatic capacity

The Minkowski problem is a characterization problem for a geometric measure generated by convex bodies: It asks for necessary and sufficient conditions in order that a given measure arises as the measure generated by a convex body. The study of Minkowski problems has a long history and strong influence on both the Brunn-Minkowski theory and fully nonlinear partial differential equations. In this talk, I will present our recent results on the Lp Minkowski problem for the capacity, which is strongly related to the PDEs and the harmonic analysis. This is a joint work with G. Xiong and L. Xu.

2019   10   12   10:45- 11:55

1114

:Sharp One Component Regularity for Naiver-Stokes

We consider the conditional regularity of mild solution v to the incompressible Naiver-Stokes equations in three dimensions. Let e S^2 and $0<T^*<$. Chemin and Zhang (Ann Sci Ec Norm Super 49: 131-167, 2016) proved the regularity of v on $(0,T^*]$ if there exists p   (4,6) such that $\left[\int_0^{T^*} \|v \cdot e\|^p_{H^{\frac{1}{2}+\frac{2}{p}}} \, dt < \infty.\right]$ Chemin et al. (Arch Ration Mech Anal 224(3): 871-905, 2017) extended the range of p to (4,   ). In this talk we settle the case p   [2,4]. Our proof also works for the case p   (4,   ). This is a joint work with Dr. Bin Han, Prof. Zhen Lei and Prof. Dong Li.

Long time asymptotics for chemotaxis with free boundary

The Patlak-Keller-Segel model can be used to model the nonlocal aggregation phenomena in the collective motion of cells or the evolution of the density of bacteria by chemotaxis. We consider the free boundary value problem for the Patlak-Keller-Segel model with the homogeneous nonlinear degenerate diffusion in this talk, which simulates the congested phenomena and the dynamical behaviors of the cells motions with finite total mass and compactly supported density distribution. For the subcritical case, we prove that the cell density function exists globally in time and tends to the corresponding steady-state at exponential time rate due to the balance between nonlinear diffusion effect and nonlocal aggregation. For the supercritical case, yet, we show that the global solution for the cell density exists and converges algebraically in time to the Barenblatt solution of the corresponding porous media equation due to the diffusion dominating mechanism.

2019   10   12   14:00- 15:45

1114

Domain Wall Solitons Arising in Classical Gauge Field Theories

Domain wall solitons are basic constructs realizing phase transitions in various field-theoretical models and are solutions to some nonlinear ordinary differential equations descending from the corresponding full sets of governing equations in higher dimensions. In this paper, we present a series of domain wall solitons arising in several classical gauge field theory models. In the context of the Abelian gauge field theory, we unveil the surprising result that the solutions may explicitly be constructed, which enriches our knowledge on integrability of the planar Liouville type equations in their one-dimensional limits. In the context of the non-Abelian gauge field

theory, we obtain some existence theorems for domain wall solutions arising in the electroweak type theories by developing some methods of calculus of variations formulated as direct and constrained minimization problems over a weighted Sobolev space.

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2019　10　13　　09:00- 10:10

1114

Exterior rigidity theorems for fully nonlinear elliptic equations

In this talk, I will discuss an exterior Bernstein type result for special Lagrangian equations with supercritical phases: solutions over exterior domains must be asymptotic to quadratic polynomials at infinity. I will also discuss quadratic asymptotic behavior of solutions of general fully nonlinear uniformly elliptic equations, of Monge-Amp\`{e}re equations (previously known), of quadratic Hessian equations, and of inverse harmonic Hessian equations over exterior domains. This is a joint work with Prof. Dongsheng Li & Prof. Yu Yuan.

On the equivalence of viscosity and distribution solutions of second-order PDEs with Neumann boundary conditions.

We apply a probabilistic approach to prove that the viscosity solutions and the distribution ones to the Neumann problem of second order elliptic and parabolic equations are equivalent.

2019　10　13　　10:40- 11:50

1114

The Free Boundary Problem in Incompressible Elastodynamics

In this talk, we show the local well-posedness of two kinds of free boundary problems in incompressible elastodynamics. When considering the two-phase problem which with different densities fluid on both sides of the free boundary, we introduce non-parallel condition of deformation tensor and prove this problem is well-posed under this condition. For elastic-vacuum model　we consider the free boundary problem with elastic fluid and vacuum on two sides of the free surface respectively, we prove that the problem is well-posed when non-parallel condition is satisfied in a part of free surface and Taylor-sign condition is satisfied in the rest part of free surface. This talk is based on joint works with Wei Wang and Zhifei Zhang.

Partial regularity of suitable weak solutions of the Navier-Stokes-Planck-Nernst-Poisson equation

In this paper, inspired by the seminal work by Caffarelli-Kohn-Nirenberg on the incompressible Navier-Stokes equation, we establish the existence of a suitable weak solution to the Navier-Stokes-Planck-Nernst-Poisson equation in dimension three, which is shown to be smooth away from a closed set whose 1-dimensional parabolic Hausdorff measure is zero.

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地图